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8. **(5 points)** `local/Library/Rochester/setLinearAlgebra16DeterminantOfTransf/ur_la_16_2.pg`

Find the determinant of the linear transformation $T : P_2 \to P_2$ given by

$$T(f) = -3f - 5f'$$

where P_2 denotes the vector space of polynomials of degree up to 2.

Hint: The determinant of T is the determinant of the matrix associated to T with respect to some basis of P_2.

$\det(T) =$ _____

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1. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi1.pg`

Are the following statements true or false?

? 1. If a linear system has four equations and seven variables, then it must have infinitely many solutions.
? 2. The linear system $A\mathbf{x} = \mathbf{b}$ will have a solution for all $\mathbf{b}$ in $\mathbb{R}^n$ as long as the columns of the matrix A span $\mathbb{R}^n$
? 3. Every linear system with free variables has infinitely many solutions.
? 4. If A and B are square matrices satisfying $\det(A) = 0$ and $\det(B) = 0$, then $A + B$ cannot be invertible.
? 5. If a linear system has the same number of equations and variables, then it must have a unique solution.

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6. **(5 points)** `local/Library/NAU/setLinearAlgebra/UpperMatrixBasisTrans2.pg`

Consider the ordered bases $B = \left(\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -2 & 3 \\ 0 & 3 \end{bmatrix} \right)$ and $C = \left(\begin{bmatrix} 1 & -3 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix} \right)$ for the vector space V of upper triangular 2×2 matrices.

a. Find the transition matrix from C to B.

$$P_{C,B} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

b. Find the coordinates of a matrix M in the ordered basis B if the coordinate vector of M in C is $[M]_C = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$.

$$[M]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

c. Find M.

$$M = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

4. **(5 points)** `local/Library/TCNJ/TCNJ_LinearIndependence/problem8.pg`

Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three linearly independent vectors in a vector space. Determine a value of k,

$k =$ ___, so that the set $S = \{\mathbf{u} - 2\mathbf{v},\ \mathbf{v} - 4\mathbf{w},\ \mathbf{w} - k\mathbf{u}\}$ is linearly dependent.

13. **(5 points)** `local/Library/Rochester/setLinearAlgebra17DotProductRn/ur_la_21.pg`

Among all unit vectors $\mathbf{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$, find the one for which the sum $x + 9y + 10z$ is minimal.

Hint: use the Cauchy-Schwarz inequality. If entering the answer as decimal numbers, make sure they are correct to four decimal places. Recall that square roots such as $\sqrt{17}$ can be typeset as sqrt(17).

$\mathbf{u} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

3. **(10 points)** `local/setSemester_A_final_assessment_2021-22/span.pg`

Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be (non-zero) vectors and suppose that $\mathbf{z} = -4\mathbf{x} - 4\mathbf{y}$ and $\mathbf{w} = -20\mathbf{x} - 20\mathbf{y} - 4\mathbf{z}$.
Are the following statements true or false?

[?] 1. Span($\mathbf{x}, \mathbf{z}$) = Span($\mathbf{x}, \mathbf{y}$)
[?] 2. Span($\mathbf{x}, \mathbf{y}, \mathbf{z}$) = Span($\mathbf{w}, \mathbf{z}$)
[?] 3. Span($\mathbf{w}, \mathbf{z}$) = Span($\mathbf{w}, \mathbf{x}$)
[?] 4. Span($\mathbf{w}, \mathbf{x}, \mathbf{y}$) = Span($\mathbf{w}, \mathbf{y}$)
[?] 5. Span($\mathbf{y}, \mathbf{z}$) = Span($\mathbf{w}, \mathbf{x}$)

11. **(7 points)** `local/Library/Rochester/setLinearAlgebra12Diagonalization/ur_la_12_1.pg`

Let

$$M = \begin{bmatrix} 12 & -10 \\ 5 & -3 \end{bmatrix}.$$

Find formulas for the entries of M^n, where n is a positive integer.

Hint: a formula such as $5 \cdot (2.3)^n + 7 \cdot (3.5)^n$ is typeset as 5*(2.3)$^n + 7 * (3.5)^n$.

$$M^n = \begin{bmatrix} \_\_\_\_\_\_\_\_\_\_\_\_ & \_\_ \\ \_\_\_\_\_\_\_\_ & \_\_ \end{bmatrix}$$

7. **(5 points)** `Library/NAU/setLinearAlgebra/shiftedSpace2.pg`

Let $V = \mathbb{R}^2$. For $(u_1, u_2), (v_1, v_2) \in V$ and $a \in \mathbb{R}$ define vector addition by $(u_1, u_2) \boxplus (v_1, v_2) := (u_1 + v_1 + 3, u_2 + v_2 - 1)$ and scalar multiplication by $a \boxdot (u_1, u_2) := (au_1 + 3a - 3, au_2 - a + 1)$. It can be shown that $(V, \boxplus, \boxdot)$ is a vector space over the scalar field $\mathbb{R}$. Find the following:

the sum:

$(9, 0) \boxplus (4, -1) =$(_____,_____)

the scalar multiple:

$0 \boxdot (9,0) =$(_____,_____)

the zero vector:

$\underline{0}_V =$(_____,_____)

the additive inverse of (x,y):

$\boxminus(x,y) =$(__________,__________)

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2. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi2.pg`

Are the following statements true or false?

[?] 1. The basis for the zero vector space $\{\mathbf{0}\}$ consists of the zero vector itself.

[?] 2. If $\{\mathbf{u},\mathbf{v},\mathbf{w}\}$ is a linearly independent set, then $\{\mathbf{u}+3\mathbf{v},\mathbf{v}-4\mathbf{w},\mathbf{w}\}$ is linearly independent.

[?] 3. If $\{\mathbf{u},\mathbf{v},\mathbf{w}\}$ is a linearly independent set, then $\{2\mathbf{u}+3\mathbf{v}+4\mathbf{w},\mathbf{u}+3\mathbf{v},\mathbf{u}+4\mathbf{w}\}$ is linearly independent.

[?] 4. There exist vectors $\mathbf{u},\mathbf{v},\mathbf{w} \in \mathbb{R}^3$ such that $\mathbf{u}-\mathbf{v},\mathbf{v}-\mathbf{w},\mathbf{w}-\mathbf{u}$ span $\mathbb{R}^3$

[?] 5. If S is a linearly independent set and T is a spanning set in a vector space V, then $S \cap T$ is a basis for V.

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9. **(8 points)** `local/Library/NAU/setLinearAlgebra/LinTransImage.pg`

Let $V = \mathbb{R}^{2\times 2}$ be the vector space of 2×2 matrices and let $L : V \to V$ be a linear transformation defined by

$$L(X) = \begin{bmatrix} -15 & -5 \\ -3 & -1 \end{bmatrix} X.$$

a. Evaluate $L(\begin{bmatrix} -3 & -4 \\ 4 & -4 \end{bmatrix}) = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

b. Find a basis for $\ker(L)$:

$$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

c. Find a basis for $\mathrm{im}(L)$:

$$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

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5. **(10 points)** `local/setSemester_A_final_assessment_2021-22/new1Problem.pg`

Determine whether the given set S is a subspace of the vector space V.

[?] 1. $V = C^3(\mathbb{R})$, and S is the subset of V consisting of those functions y satisfying the differential equation $y''' - y' = 1$.

[?] 2. $V = P_n$, and S is the subset of P_n consisting of those polynomials satisfying $p(0) = 0$.

[?] 3. $V = \mathbb{R}^{n\times n}$, and S is the subset of all matrices A satisfying $A^T = -A$.

[?] 4. $V = \mathbb{R}^n$, and S is the set of solutions to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ where A is a fixed $m \times n$ matrix.

[?] 5. $V = \mathbb{R}^{n\times n}$, and S is the subset of all upper triangular matrices.

Notation: P_n is the vector space of polynomials of degree up to n, and $C^n(\mathbb{R})$ is the vector space of n times continuously differentiable functions on $\mathbb{R}$.

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12. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi4.pg`

Are the following statements true or false?

[?] 1. If $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \|\mathbf{u}-\mathbf{v}\|^2$, then the vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

[?] 2. If $\mathbf{x}$ is not in a subspace W, then $\mathbf{x} - \mathrm{proj}_W(\mathbf{x})$ is zero.

[?] 3. The best approximation to $\mathbf{y}$ by elements of a subspace W is given by the vector $\mathbf{y} - \mathrm{proj}_W(\mathbf{y})$.

? 4. If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for W, then multiplying $\mathbf{v}_3$ by a non-zero scalar c gives a new orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, c\mathbf{v}_3\}$.

? 5. If an $n \times p$ matrix U has orthonormal columns, then $UU^T\mathbf{x} = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$.

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10. **(10 points)** `local/setSemester_A_final_assessment_2021-22/multi3.pg`

Are the following statements true or false for a square matrix A?

? 1. A number c is an eigenvalue of A if and only if the equation $(A - cI)\mathbf{x} = \mathbf{0}$ has a nontrivial solution $\mathbf{x}$.

? 2. To find the eigenvalues of A, reduce A to echelon form.

? 3. If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$ and some scalar λ, then λ is an eigenvalue of A.

? 4. If an $n \times n$ matrix A is diagonalizable, then A has n distinct eigenvalues.

? 5. If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$ and some scalar λ, then $\mathbf{x}$ is an eigenvector of A.