Queen Mary
University of London

Main Examination period 2021 – January – Semester A

MTH6154 / MTH6154P: Financial Mathematics I

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, **you must submit within the first 3 hours of the assessment period**.

Examiners: Dr. D. Stark, Dr. K. Glau

Question 1 [15 marks]. Mile End Bank offers both deposits and loans at interest rate $r = 2.5\%$ compounded semi-annually.

(a) If you deposit £$(200 + b)$ in a Mile End Bank account, where b is the last digit of your student number, and make no withdrawals, how much money will be in the account 6 years later? State your answer to the nearest pence. **[3]**

(b) What is the effective rate offered by Mile End Bank? State your answer as a percentage to three significant figures. **[3]**

East London Bank offers an account with interest that is continuously compounded at rate 7% for the first year and 5% for the second year, after which the interest rate is continuously compounded at rate 1%.

(c) Sketch the graph of the interest rate $r(t)$ as a function of years t. **[3]**

(d) Find the yield curve $\bar{r}(t)$ for all $t > 0$. **[6]**

Question 2 [22 marks].

Suppose interest is compounded continuously at rate 2%. A company has a liability of £100, 000 of 4-year zero coupon bonds. It wants to Reddington immunise this liabilities with the purchase of P 3-year zero coupon bonds and Q 5-year zero coupon bonds, where P and Q are to be determined.

(a) Derive the two equations P and Q must satisfy. **[6]**

(b) Solve the equations for P and Q. State P and Q to the nearest pound. **[6]**

(c) Explain the meaning of Redding immunisation and how it is used in practice in your own words. Do not use more than 60 words. **[10]**

Question 3 [12 marks]. Suppose you decide to model interest rates by using the Varying Interest Rate model where R_i is the effective interest rate for a deposit between time i and time $i + 1$ and the R_i are an i.i.d. sequence $\{R_i\}_{i=0,1,2,...}$ of random variables with common distribution

$$R_i = \begin{cases} 5\% & \text{with probability } 0.4, \\ 6\% & \text{with probability } 0.6. \end{cases}$$

(a) If £100 is deposited at time $i = 0$, then what is the expected value of the deposit at time $i = 4$? Derive the result and then state it to the nearest pence. **[5]**

(b) Suppose you deposit £100 at time $i = 0$ and another £100 at time $i = 1$. Determine the expected value of the money accumulated at time $i = 2$. Derive the result and then state it to the nearest pence. **[7]**

Question 4 [16 marks].

For this problem assume the spot rate curve

$$s_1 = 2\%,\ s_2 = 2.5\%,\ s_3 = 3\%,\ s_4 = 3.5\%$$

is valid.

(a) Assuming no-arbitrage, calculate the fair forward rate $f_{1,4}$. State your answer as a percentage to three significant figures. **[4]**

(b) Using the spot rate curve, find the no-arbitrage price of a 4-year bond with face value £200, 000 with annual coupons at rate 3% and redeemable at par. State your answer to the nearest pound. **[5]**

(c) Supposing the market price of the bond in (b) was lower than the no-arbitrage price you found, explain how you would you construct an arbitrage. **[7]**

Question 5 [8 marks]. Consider the upcoming England vs Montenegro football match, for which a bookmaker is quoting the following odds:

Outcome	Odds
England win	3/1
Draw	3/1
Montenegro win	4/1

(a) By doing a calculation with the odds, show that an arbitrage opportunity exists. **[3]**

(b) A betting strategy is a vector (x, y, z), where £x is bet on England winning, £y is bet on a draw, and £z is bet on Montenegro winning. Show that $(1, 1, 2)$ is betting strategy giving an arbitrage opportunity. **[5]**

Question 6 [27 marks]. A share price is modelled via a two-period binomial model with initial stock price $S = 208$, up/down multiplication factors $u = 7/4$ and $d = 1/4$, and interest rate per time-period $r = 4.5\%$.

(a) Verify that the no-arbitrage assumption is valid in this model. **[4]**

(b) Find the risk-neutral probabilities of up and down movements in the share price. State your answers to two significant figures. **[4]**

(c) Find the no-arbitrage price of a European put option on the share with strike $K = 190$ and expiry date $T = 2$. State your answer to the nearest pence. **[10]**

(d) Suppose that the share goes up in the first time step from time 0 to time 1. Find the replicating portfolio for the put option from (c) that will be used at time 1. State your answers to three significant digits. **[6]**

(e) Suppose that the share goes up in the first time step. Determine whether an American put option with strike price $K = 190$ and expiry date $T = 2$ will be exercised at time $t = 1$. **[3]**

End of Paper.