Main Examination period 2022 – May/June – Semester B

MTH6139: Time Series

> You should attempt ALL questions. Marks available are shown next to the questions.

> **In completing this assessment:**
>
> - **You may use books and notes.**
>
> - **You may use calculators and computers, but you must show your working for any calculations you do.**
>
> - **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
>
> - **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;

- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. To be eligible for IFoA exemption, **you must submit your exam within the first 3 hours of the 24-hour exam period**.

Examiners: P. Bhuyan, C. Sutton

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Question 1 [15 marks].

(a) Show that if

$$\sum_{j=-\infty}^{\infty} a_j = 1, \quad \sum_{j=-\infty}^{\infty} ja_j = 0, \quad \sum_{j=-\infty}^{\infty} j^2 a_j = 0,$$

then for $m_t = \beta_0 + \beta_1 t + \beta_2 t^2$, we have

$$m_t = \sum_{j=-\infty}^{\infty} a_j m_{t-j}.$$

[7]

(b) Suppose we are given the linear filter

$$a_0 = \frac{74}{320}, a_1 = \frac{67}{320}, a_2 = \frac{46}{320}, a_3 = \frac{21}{320}, a_4 = \frac{3}{320}, a_5 = -\frac{5}{320}, a_6 = -\frac{6}{320}, a_7 = -\frac{3}{320}$$

with $a_j = a_{-j}$ for $-7 \leq j \leq 7$. Does the quadratic trend $m_t = \beta_0 + \beta_1 t + \beta_2 t^2$ pass through the filter without distortion? [8]

Question 2 [21 marks].

(a) Consider the additive model $X_t = m_t + Y_t$ with cubic trend
$m_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$.

   (i) Show that ∇m_t is a polynomial of degree 2, i.e., a quadratic function. [4]

   (ii) What is $\nabla^4 m_t$? Show your reasoning. [7]

(b) Exponential smoothing is defined by the following recursion:

$$\widehat{m}_1 = X_1,$$
$$\widehat{m}_t = \alpha X_t + (1-\alpha)\widehat{m}_{t-1}, \quad t = 2, \ldots, n.$$

Suppose we have time series data $\{7, 4, 1, 3, 8, 11, 11\}$ with $X_1 = 7, X_2 = 4$ and so on until $X_7 = 11$. Assume that $\alpha = 0.6$. Calculate $\widehat{m}_6$ using exponential smoothing. [10]

Question 3 [21 marks]. Consider the following AR(1) model

$$X_t = 0.5X_{t-1} + Z_t,$$

where $\{Z_t\}$ is a white noise process, that is, a sequence of uncorrelated random variables with mean zero and constant variance 1.

(a) Define what it means for an autoregressive process to be **causal**, and state whether the AR(1) process above is causal or not. [4]

(b) Develop expressions for the autocovariance and autocorrelation functions for this process. **[6]**

(c) Write down the partial autocorrelation functions for this process. **[2]**

(d) Using the autocovariance function from part (b), compute the variance of the sample mean $(X_1 + X_2 + X_3)/3$. **[9]**

Question 4 [20 marks]. Consider the time series

$$X_t + 0.3X_{t-1} - 0.1X_{t-2} = Z_t - 0.9Z_{t-1} + 0.14Z_{t-2},$$

where Z_t is a white noise random variable.

(a) Determine the values of p and q in the standard $\text{ARMA}(p, q)$ framework so that there is no parameter redundancy in the model. **[6]**

(b) Check whether the process is causal and invertible. **[4]**

(c) Obtain a linear process form of the AMRA model identified in part (a). **[10]**

Question 5 [23 marks]. Figure 1 below shows monthly totals of international airline passengers, 1949 to 1960.

(a) Describe the main features of the time series shown by the plot in Figure 1. **[6]**

(b) What model do you consider to combine different features present in this time series data? **[2]**

(c) What possible analyses could you do, using a suitable computer package such as `R`, to determine the best model for forecasting purpose? **[11]**

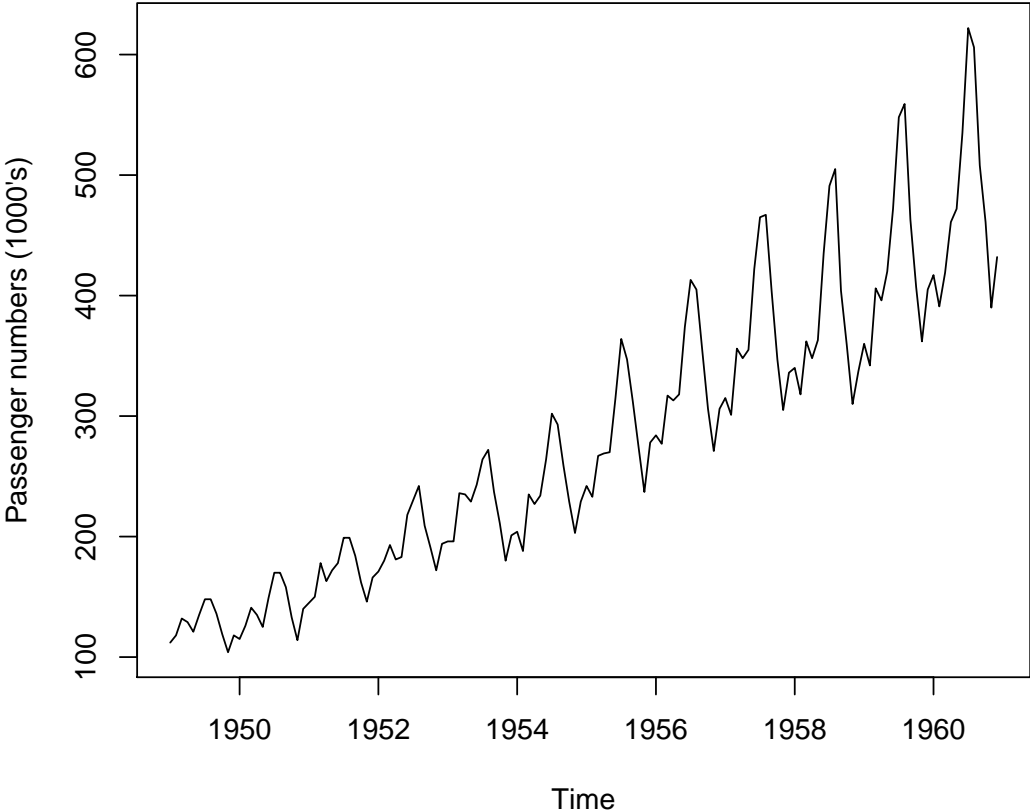(d) Discuss the diagnostic tests to check whether the model fits well. **[4]**

End of Paper.

Figure 1: Airline Passenger Data