Queen Mary
University of London

Main Examination period 2018

MTH6139: Time Series

Duration: 2 hours

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Examiners: L. Pettit, J. Griffin

Question 1. [24 marks]
Let $\{X_t\}$ be a time series such that

$$X_t = m_t + Y_t$$

where m_t denotes a polynomial trend and Y_t denotes a zero-mean weakly stationary process.

(a) (i) Define what it means for a process to be **weakly stationary**. **[3]**

(ii) Define what it means for a process to be **strictly stationary**. **[3]**

(iii) Are all weakly stationary processes also strictly stationary? **[1]**

(b) Define the operator ∇ and explain how it can be used to remove the trend from the time series $\{X_t\}$. **[3]**

(c) Show that $\nabla^2 X_t$ is a weakly stationary process when the trend is quadratic and give its autocovariance function in terms of the process Y_t. **[8]**

(d) Explain what is meant by the convolution operation on the linear filters$\{a_j\}$ and $\{b_k\}$. Show that the operator ∇^2 is a convolution of two filters of the form $(-1, 1)$. **[6]**

Question 2. [16 marks] An AR(1) process with parameter ϕ is defined by the equation

$$X_t = \phi X_{t-1} + Z_t,$$

where $\{Z_t\}$ is a white noise process, that is, a sequence of uncorrelated random variables with mean zero and constant variance σ^2.

(a) Define what it means for an autoregressive process to be **causal**. **[2]**

(b) Show that an AR(1) process is causal if the parameter ϕ satisfies a condition which you should state. **[6]**

(c) The autocovariance of X_t and $X_{t+\tau}$ is defined to be $\gamma(\tau)$. For the AR(1) process with parameter ϕ find $\gamma(0)$ and $\gamma(1)$ and write down $\gamma(\tau)$ for $\tau \geq 2$. **[6]**

(d) Hence calculate the autocorrelation function (ACF) for the AR(1) process. **[2]**

Question 3. [12 marks] Explain what plots would be expected for the autocorrelation function (ACF) and partial autocorrelation function (PACF) for the following processes.

(a) White noise **[3]**

(b) AR(2) **[3]**

(c) MA(1) **[3]**

(d) ARMA(1,2). **[3]**

Question 4. [26 marks]

(a) Consider the following three time series models where the error terms Z_t are uncorrelated random errors with zero mean and constant variance.

(i)
$$X_t - 1.2X_{t-1} = Z_t.$$

(ii)
$$X_t - 1.2X_{t-1} + 0.2X_{t-2} = Z_t - 0.7Z_{t-1}.$$

(iii)
$$X_t = Z_t + 0.7Z_{t-1} - 0.5Z_{t-2}$$

For each model, say whether it is stationary or not and specify p and q in the standard ARMA (p, q) framework. **[8]**

(b) Consider the time series

$$X_t + 0.3X_{t-1} - 0.1X_{t-2} = Z_t - 0.9Z_{t-1} + 0.14Z_{t-2},$$

where Z_t is a white noise random variable.

(i) Determine the values of p and q in the standard ARMA(p, q) framework so that there is no parameter redundancy in the model. **[5]**

(ii) Check whether the process is causal and invertible. **[4]**

(iii) Obtain a linear process form of this time series. **[9]**

Question 5. [22 marks]
A sports retailer recorded how many sets of golf clubs he sold from his shops each month for 4 years. The data is shown in the plot on the next page. The retailer would like you to forecast how many sets of clubs he can expect to sell in each of the next three months.

(a) Describe the main features of the time series shown by the plot. **[4]**

(b) What possible analyses could you do, using a suitable computer package such as MINITAB, to determine a stationary time series model for the data? Include brief details of diagnostic tests to check if the model is fitting well. **[10]**

(c) How could you forecast the sales for the next three months? **[4]**

(d) The retailer then tells you that the number of shops he sold from varied over the four year period. If you had the data on the number of shops open each month how might you change your analysis? What assumptions might be reasonable? **[4]**

End of Paper.


Time Series Plot of Sales
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