Queen Mary
University of London

# B. Sc. Examination by course unit 2014

## MTH 6139 Time Series

**Duration: 2 hours**

**Date and time: 2 June 2014, 10.00-12.00**

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You should attempt all questions. Marks awarded are shown next to the questions.

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Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): D. S. Coad

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**Question 1 (17 marks)** Let $\{X_t\}_{t=1,2,\ldots}$ be a time series such that

$$X_t = m_t + Y_t,$$

where $m_t$ denotes a polynomial trend of degree $k$ and $Y_t$ denotes a zero-mean weakly stationary process.

(a) Define the operator $\nabla$ and explain how it can be used to remove the trend from the time series $\{X_t\}$. [3]

(b) Show that $\nabla^2 X_t$ is a weakly stationary process when the trend is quadratic and give its autocovariance function in terms of the process $Y_t$. [8]

(c) Explain what is meant by the convolution operation on the linear filters $\{a_j\}$ and $\{b_k\}$. Show that the operator $\nabla^2$ is a convolution of two filters of the form $(-1, 1)$. [6]

**Question 2 (17 marks)** Consider the MA($q$) process for a time series $\{X_t\}$ given by

$$X_t = Z_t + \theta_1 Z_{t-1} + \ldots + \theta_q Z_{t-q},$$

where $\{Z_t\} \sim WN(0, \sigma^2)$.

(a) Give the definition of a causal process. [2]

(b) By expressing this time series in the form of a linear process, show that its autocovariance function is

$$\gamma(\tau) = \begin{cases} \sigma^2 \sum_{j=0}^{q-|\tau|} \theta_j \theta_{j+|\tau|} & \text{if} \quad |\tau| \leq q, \\ 0 & \text{if} \quad |\tau| > q. \end{cases}$$

What is the corresponding autocorrelation function? [12]

(c) Explain why this MA($q$) process is a weakly stationary series. [3]

**Question 3 (25 marks)** Let the ARMA$(1, 2)$ process for a time series $\{X_t\}$ be given by

$$X_t - 0.9 X_{t-1} = Z_t + 0.3 Z_{t-1} - 0.4 Z_{t-2},$$

where $\{Z_t\} \sim WN(0, \sigma^2)$.

(a) Write down the operator form of this process. Show that it is invertible. [8]

(b) Obtain the linear process form of this time series. [12]

(c) State the difference equations for the autocovariance function of this ARMA$(1, 2)$ process. How does this function behave for the process? [5]

**Question 4 (24 marks)** Consider an AR(2) process of the form

$$X_t = 0.9X_{t-1} - 0.2X_{t-2} + Z_t,$$

where $\{Z_t\} \sim WN(0, \sigma^2)$.

(a) Show that there is a stationary solution to this process. [6]

(b) Using the difference equations, obtain the autocorrelation function for this process. [16]

(c) Give the partial autocorrelation function for this process. [2]

**Question 5 (17 marks)** Suppose that an ARIMA$(p, d, q)$ model is to be fitted to some time series data $\{x_t\}_{t=1,\ldots,n}$.

(a) Describe what important features of the data can be revealed by a time series plot. [6]

(b) Explain how the orders $p$, $d$ and $q$ can be identified. [5]

(c) Having fitted an ARIMA$(p, d, q)$ model to the data, what residual diagnostics should be looked at, and why? [6]

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**End of Paper**