Queen Mary
University of London

Main Examination period 2017

MTH6120
Further Topics in Mathematical Finance

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: Dr. S. Sarfo

Question 1. [25 marks]

(a) An investor has the opportunity to choose among banks A, B and C to deposit £1000 for 6 months. Each bank offers a different time-dependent interest rate.

Bank A offers to pay continuous interest on the savings with the time dependent interest rate $r_A(t) = (1 + \sin(\pi t))/100$,

Bank B offers $r_B(t) = \exp(t/10)/80$,

and bank C with the constant rate $r_c(t) = 1/68$ where t denotes time in units of years in all cases.

Which bank should the investor choose in order to maximise the amount of money in her account? Justify your answer. [**5**]

(b) (i) State the forward price F_T at time T of an asset currently traded at S_0, if the market offers a continuously compounded interest rate r? [**3**]

(ii) state the Call-Put parity formula. [**3**]

(iii) Consider a European call option and a European put option on a non-dividend-paying stock. You are given the current price of the stock as £60 and that the call option currently sells for £0.15 more than the put option. Both the call option and put option will expire in 4 years and both the call option and put option have a strike price of £70. Calculate the continuously compounded risk-free interest rate. [**3**]

(c) Give the criteria for a function $u(x)$ to be a utility function.
Show whether or not the each of the following functions satisfy the utility criteria and if so give the range. [**3**]

(i) $u(x) = 17 + x + \log(x + a)$ [**4**]

(ii) $u(x) = 3 - e^{-bx}$ [**4**]

Question 2. [25 marks]

(a) (i) State the formula of the capital asset pricing model, **CAPM**. [**2**]

(ii) Explain the meaning of **systemic risk** and **idiosyncratic risk** in CAPM. [**3**]

(b) Derive the variance $Var(R_i)$ of the return of an investment i under the CAPM? [**7**]

(c) Consider two securities with different expectations of their rate of returns r_1, r_2 and different variances v_1^2, v_2^2. Suppose that an investor has a utility function $u(x) = 1 - \mathrm{e}^{(a-bx)}$ where a and b are constants with $b > 0$.

Derive the formulas for the optimal portfolio weights (w_1, w_2) invested in the two assets for general parameters a and $b > 0$. [**7**]

(d) Under the assumptions of the CAPM, suppose that the current risk-free interest rate is 6% and that the expected value and standard deviation of the market rate of return are 10% and 20% respectively. If the covariance of the rate of return of a given stock and the market's rate of return is 0.05, what is the expected rate of return of that stock? (Assume the investment period is one year). [**6**]

Question 3. [25 marks]

(a) State any 3 differences between a futures contract and an option contract. [**3**]

(b) Briefly explain what each of the following sensitivity values mean assuming a call option:

(i) $\text{Delta} = 0.522$ [**2**]

(ii) $\text{Rho} = 0.0891$ [**2**]

(c) Consider a European call option with a strike price of £60 which costs £10. Draw a graph illustrating the net payoff of the option for stock prices in the interval $[0, 100]$, ignoring the time value of money. [**3**]

(d) Why are financial assets better modelled by Geometric Brownian Motion than by Brownian Motion? [**4**]

(e) Describe the relationship between the **no arbitrage** principle of financial markets with the risk-free interest. [**3**]

(f) If a share of a security is currently traded at £31, a three-month European call option is £3, with a strike price of £31 and the risk-free interest rate is 10%

(i) What is the arbitrage-free European put option price for the security? [**3**]

(ii) If the European put option price for the security were £3, what arbitrage strategy will be opened to an investor? [**5**]

Question 4. [25 marks]

(a) Define the following terms:

(i) **Zero coupon bond**. [3]

(ii) **Yield to maturity**. [3]

(b) Define and state the formula for **effective duration** and **convexity** of a cashflow $A(r)$ taking place at time T, where r is the interest rate. Assume that interest rate is continuously compounded. [4]

(c) Consider a 5 year bond with a face value of $ 100 that pays an annual coupon of 8% . Assume spot rates are flat at 5%

Find the bond's price and duration. [4]

(d) (i) Write down the three conditions for the Redington's immunisation. [3]

(ii) Briefly explain any two limitations of the classical immunisation theory. [2]

(e) Consider a series of annual investments, each of amount 1, at the beginning of years $1, 2, \ldots, n$.

At the end of year n the value A_n of this cash flow under random interest rates can be written as the recurrence relation

$$A_n = (1 + r_n)(1 + A_{n-1})$$

where r_k is the random interest rate of the kth year, with mean μ and variance γ^2, for all $1 \leq k \leq n$ and where $r_1, r_2, \ldots, r_n$ are independent.

Derive recurrence relations for the mean $\mathbb{E}(A_n)$ and the second moment $\mathbb{E}(A_n^2)$. [6]

End of Paper.