

Main Examination period 2017

MTH6100/MTH6100P

Actuarial Mathematics

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

Only non-programmable calculators that have been approved from the college list of non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: J.Webber, W.Just

Question 1. [20 marks] A loan of £2,500 will be repaid after 90 days by paying £2,583.

- (a) What is the annual simple rate of interest that the borrower is charged on the loan of £2,500? [4]
- (b) What is the Annual Percentage Rate (APR) that the borrower is charged on the loan of £2,500? [4]
- (c) If the APR is unchanged from that calculated in (b) above, find the amount outstanding after 5 years if the loan is not repaid. [4]
- (d) Assuming the APR is 12%, determine the annual rate of interest convertible four times per year. [4]
- (e) Assuming the APR is 12%, determine the annual rate of discount convertible four times per year. [4]

Question 2. [20 marks] In the state of Broxidge, students leave university with debts of £20,000 each. Interest is payable on these loans at an annual effective rate of 4.6% per year.

- (a) Calculate the monthly payment, made in arrears, required to exactly pay off the loan in 30 years. [4]
- (b) A student has been repaying the loan at the rate of £200 per month in arrears for 5 years. Calculate the loan outstanding after 5 years. [4]
- (c) Another student makes repayments of £150 per month in arrears.
 - (i) Calculate the period of time until the loan is repaid. [7]
 - (ii) Find the exact amount of the last monthly payment to the nearest penny. [5]

Question 3. [26 marks]

- (a) Describe in words the actuarial symbol ${}_{u|t}q_x$. [4]
- (b) Calculate ${}_{10|5}q_{20}$ using the ELT17 Male mortality table. [2]

The remaining parts of this question use the survival function $s(x) = (1 - x/100)^3$ for $0 \leq x \leq 100$.

- (c) Calculate ${}_{10|5}q_{20}$. [6]
- (d) Derive an expression for the force of mortality, $\mu(x)$, assuming the same survival function. [4]
- (e) Calculate the complete expectation of life at age 3.5 using this survival function. [6]
- (f) Calculate the number of deaths each year, given this survival function and a stationary population of 650,000. [4]

Question 4. [34 marks]

- (a) Show that $A_{x:\overline{n}|} = 1 - d\ddot{a}_{x:\overline{n}|}$ where d is the annual rate of discount. [6]
- (b) Calculate the annual premium payable monthly in advance for a £15,000 endowment assurance of term 30 years for a policyholder aged 40 exactly. Payments are made at the end of the policy year. Assume interest of 4% per annum and mortality given by the AMC00 Select table. [8]
- (c) Derive, with explanation, the reserve per policy in force at time t ($t < n$) for an n year endowment assurance of unit sum insured taken out by a life aged x with premiums paid annually in advance. Express the reserve in terms of life annuity factors. Assume that the next premium is immediately payable. [6]
- (d) An insurer sells 500 policies of the type given in part (b) above on 1st June 2016, all to 40 year old men. Calculate the expected total reserves for this policy type after 15 years, assuming that the next premium is immediately payable. [8]
- (e) Calculate the variance of the expected amount of reserves after 15 years. [6]

End of Paper—An appendix of 8 pages follows.

English Life Tables No 17 - Males

Age x	p_x	q_x	l_x	d_x	μ_x	e_x
0	0.995254	0.004746	100000	475		78.97
1	0.999694	0.000306	99525	30	0.000369	78.34
2	0.999793	0.000207	99495	21	0.000246	77.37
3	0.999853	0.000147	99474	14	0.000172	76.38
4	0.999885	0.000115	99460	12	0.000128	75.39
5	0.999901	0.000099	99448	10	0.000105	74.40
6	0.999909	0.000091	99438	9	0.000094	73.41
7	0.999912	0.000088	99429	9	0.000089	72.42
8	0.999912	0.000088	99420	8	0.000088	71.42
9	0.999911	0.000089	99412	9	0.000088	70.43
10	0.999909	0.000091	99403	9	0.000090	69.43
11	0.999906	0.000094	99394	9	0.000092	68.44
12	0.999900	0.000100	99385	10	0.000097	67.45
13	0.999888	0.000112	99375	12	0.000105	66.45
14	0.999866	0.000134	99363	13	0.000121	65.46
15	0.999828	0.000172	99350	17	0.000150	64.47
16	0.999767	0.000233	99333	23	0.000200	63.48
17	0.999689	0.000311	99310	31	0.000271	62.50
18	0.999609	0.000391	99279	39	0.000352	61.52
19	0.999544	0.000456	99240	45	0.000427	60.54
20	0.999504	0.000496	99195	49	0.000480	59.57
21	0.999484	0.000516	99146	52	0.000508	58.60
22	0.999471	0.000529	99094	52	0.000523	57.63
23	0.999463	0.000537	99042	53	0.000534	56.66
24	0.999457	0.000543	98989	54	0.000540	55.69
25	0.999450	0.000550	98935	54	0.000545	54.72
26	0.999434	0.000566	98881	56	0.000556	53.75
27	0.999406	0.000594	98825	59	0.000578	52.78
28	0.999366	0.000634	98766	62	0.000612	51.81
29	0.999318	0.000682	98704	68	0.000657	50.84
30	0.999272	0.000728	98636	71	0.000706	49.87
31	0.999237	0.000763	98565	76	0.000747	48.91
32	0.999205	0.000795	98489	78	0.000779	47.95
33	0.999162	0.000838	98411	82	0.000815	46.98
34	0.999105	0.000895	98329	88	0.000864	46.02
35	0.999031	0.000969	98241	96	0.000929	45.06
36	0.998941	0.001059	98145	103	0.001012	44.11
37	0.998837	0.001163	98042	114	0.001110	43.15
38	0.998727	0.001273	97928	125	0.001218	42.20
39	0.998622	0.001378	97803	135	0.001327	41.26

English Life Tables No 17 – Males

Age x	p_x	q_x	l_x	d_x	μ_x	e_x
40	0.998527	0.001473	97668	144	0.001427	40.31
41	0.998425	0.001575	97524	153	0.001522	39.37
42	0.998296	0.001704	97371	166	0.001637	38.43
43	0.998144	0.001856	97205	181	0.001780	37.50
44	0.997985	0.002015	97024	195	0.001937	36.57
45	0.997832	0.002168	96829	210	0.002095	35.64
46	0.997689	0.002311	96619	223	0.002243	34.72
47	0.997548	0.002452	96396	237	0.002383	33.79
48	0.997381	0.002619	96159	252	0.002533	32.88
49	0.997165	0.002835	95907	272	0.002722	31.96
50	0.996895	0.003105	95635	297	0.002965	31.05
51	0.996571	0.003429	95338	326	0.003264	30.15
52	0.996209	0.003791	95012	361	0.003612	29.25
53	0.995831	0.004169	94651	394	0.003984	28.36
54	0.995424	0.004576	94257	432	0.004374	27.47
55	0.994952	0.005048	93825	473	0.004813	26.60
56	0.994417	0.005583	93352	521	0.005323	25.73
57	0.993866	0.006134	92831	570	0.005875	24.87
58	0.993313	0.006687	92261	617	0.006426	24.02
59	0.992701	0.007299	91644	669	0.007004	23.18
60	0.991984	0.008016	90975	729	0.007675	22.35
61	0.991227	0.008773	90246	792	0.008430	21.52
62	0.990508	0.009492	89454	849	0.009174	20.71
63	0.989741	0.010259	88605	909	0.009904	19.90
64	0.988773	0.011227	87696	984	0.010766	19.10
65	0.987596	0.012404	86712	1076	0.011861	18.32
66	0.986331	0.013669	85636	1171	0.013109	17.54
67	0.984992	0.015008	84465	1267	0.014416	16.78
68	0.983405	0.016595	83198	1381	0.015877	16.02
69	0.981462	0.018538	81817	1517	0.017674	15.29
70	0.979309	0.020691	80300	1661	0.019789	14.56
71	0.977140	0.022860	78639	1798	0.022010	13.86
72	0.974919	0.025081	76841	1927	0.024243	13.17
73	0.972521	0.027479	74914	2059	0.026590	12.50
74	0.969807	0.030193	72855	2199	0.029194	11.84
75	0.966638	0.033362	70656	2357	0.032210	11.19
76	0.962948	0.037052	68299	2531	0.035757	10.56
77	0.958806	0.041194	65768	2709	0.039830	9.95
78	0.954214	0.045786	63059	2888	0.044363	9.35
79	0.948888	0.051112	60171	3075	0.049514	8.78

AMC00 Male Life Table

Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x	Age x	$I_{[x]}$	$I_{[x-1]+1}$	I_x	Age x
17	9997.5094		10000.0000	17	59	9464.7096	9478.8531	9480.7305	59
18	9992.9305	9994.6901	9995.4200	18	60	9410.6849	9427.0212	9429.5820	60
19	9988.3337	9990.1025	9990.8321	19	61	9350.1805	9369.0144	9372.4010	61
20	9983.6991	9985.4871	9986.2163	20	62	9282.4873	9304.1589	9308.5187	62
21	9979.0567	9980.8438	9981.5827	21	63	9206.8320	9231.7028	9237.1968	63
22	9974.3666	9976.1827	9976.9213	22	64	9122.3956	9150.8360	9157.6369	64
23	9969.6487	9971.4740	9972.2222	23	65	9028.3257	9060.7100	9069.0001	65
24	9964.8832	9966.7276	9967.4854	24	66	8923.7129	8960.4327	8970.3747	66
25	9960.0701	9961.9336	9962.7010	25	67	8807.6361	8849.0571	8860.8106	67
26	9955.1996	9957.0921	9957.8691	26	68	8679.1539	8725.6105	8739.3111	68
27	9950.2618	9952.1832	9952.9698	27	69	8537.3365	8589.1250	8604.8568	69
28	9945.2469	9947.1972	9947.9933	28	70	8381.2654	8438.6022	8456.4058	70
29	9940.1847	9942.1340	9942.9398	29	71	8210.1056	8273.0800	8292.9181	71
30	9935.0655	9936.9939	9937.7794	30	72	8023.0608	8091.6666	8113.3848	72
31	9929.8794	9931.7671	9932.5024	31	73	7819.4936	7893.5124	7916.8461	73
32	9924.5767	9926.4238	9927.0892	32	74	7598.8879	7677.9138	7702.4263	74
33	9919.1182	9920.9344	9921.5201	33	75	7360.9680	7444.3113	7469.3894	75
34	9913.4841	9915.2596	9915.7755	34	76	7105.6585	7192.3651	7217.1705	76
35	9907.6152	9909.3799	9909.8162	35	77	6833.2053	6921.9772	6945.4296	77
36	9901.4922	9903.2460	9903.6126	36	78	6543.2750	6633.3614	6654.1244	78
37	9895.0760	9896.8385	9897.1357	37	79	6234.6920	6326.2215	6343.5631	79
38	9888.3275	9890.1087	9890.3363	38	80	5905.8869	6000.3050	6014.4463	80
39	9881.1880	9882.9977	9883.1559	39	81	5557.1270	5655.4241	5667.9541	81
40	9873.5893	9875.4570	9875.5558	40	82	5191.0860	5293.5691	5305.7888	82
41	9865.4534	9867.4085	9867.4578	41	83	4811.5773	4917.5469	4930.2080	83
42	9856.7223	9858.7745	9858.7942	42	84	4422.4381	4531.0383	4544.0494	84
43	9847.3090	9849.4875	9849.4875	43	85	4027.2597	4137.4915	4150.7347	85
44	9837.1169	9839.4312	9839.4312	44	86	3630.1730	3740.8772	3754.2191	86
45	9826.0499	9828.5291	9828.5291	45	87	3235.7358	3345.6473	3358.9374	87
46	9813.9735	9816.6562	9816.6562	46	88	2848.8406	2956.5986	2969.6769	88
47	9800.7439	9803.6786	9803.6786	47	89	2474.5254	2578.7306	2591.4291	89
48	9786.2090	9789.4437	9789.4437	48	90	2117.7709	2217.0436	2229.1940	90
49	9770.1885	9773.7708	9773.7708	49	91		1876.3069	1887.7528	91
50	9752.4550	9756.4712	9756.4712	50	92			1571.4202	92
51	9732.8031	9737.3192	9737.3192	51	93			1283.8047	93
52	9710.9228	9716.0627	9716.0627	52	94			1027.5920	94
53	9686.5083	9692.4138	9692.4332	53	95			804.3661	95
54	9659.2100	9666.0213	9666.1182	54	96			614.5164	96
55	9628.6170	9636.5205	9636.7719	55	97			457.2229	97
56	9594.3554	9603.4960	9604.0069	56	98			330.5502	98
57	9555.9547	9566.5317	9567.3964	57	99			231.6268	99
58	9512.9284	9525.1558	9526.4767	58	100			156.9026	100

AMC00 Male Life Functions (4%)

Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x	Age x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	Age x
17	5132.45		5133.73	17	59	935.71	937.11	937.29	59
18	4932.79	4933.66	4934.02	18	60	894.58	896.14	896.38	60
19	4740.89	4741.73	4742.07	19	61	854.65	856.37	856.68	61
20	4556.43	4557.25	4557.58	20	62	815.83	817.73	818.11	62
21	4379.15	4379.93	4380.25	21	63	778.05	780.16	780.62	63
22	4208.74	4209.50	4209.82	22	64	741.27	743.58	744.13	64
23	4044.95	4045.69	4045.99	23	65	705.41	707.94	708.59	65
24	3887.51	3888.23	3888.53	24	66	670.42	673.18	673.92	66
25	3736.19	3736.89	3737.18	25	67	636.25	639.24	640.09	67
26	3590.73	3591.42	3591.70	26	68	602.85	606.08	607.03	68
27	3450.92	3451.58	3451.85	27	69	570.19	573.65	574.70	69
28	3316.52	3317.17	3317.43	28	70	538.24	541.92	543.07	70
29	3187.33	3187.96	3188.22	29	71	506.97	510.86	512.08	71
30	3063.17	3063.76	3064.00	30	72	476.36	480.44	481.73	72
31	2943.81	2944.37	2944.59	31	73	446.42	450.65	451.98	73
32	2829.08	2829.61	2829.80	32	74	417.14	421.48	422.82	74
33	2718.77	2719.27	2719.43	33	75	388.54	392.94	394.26	75
34	2612.72	2613.19	2613.32	34	76	360.64	365.04	366.30	76
35	2510.74	2511.19	2511.30	35	77	333.47	337.80	338.95	77
36	2412.68	2413.11	2413.20	36	78	307.04	311.27	312.24	78
37	2318.39	2318.80	2318.87	37	79	281.31	285.44	286.22	79
38	2227.70	2228.10	2228.15	38	80	256.22	260.32	260.93	80
39	2140.47	2140.86	2140.90	39	81	231.82	235.92	236.44	81
40	2056.56	2056.95	2056.97	40	82	208.22	212.33	212.82	82
41	1975.83	1976.22	1976.23	41	83	185.58	189.66	190.15	83
42	1898.16	1898.55	1898.56	42	84	164.01	168.03	168.52	84
43	1823.41	1823.81	1823.81	43	85	143.61	147.54	148.01	85
44	1751.46	1751.87	1751.87	44	86	124.47	128.26	128.72	86
45	1682.20	1682.63	1682.63	45	87	106.68	110.30	110.74	87
46	1615.52	1615.96	1615.96	46	88	90.31	93.73	94.14	88
47	1551.29	1551.75	1551.75	47	89	75.43	78.60	78.99	89
48	1489.41	1489.90	1489.90	48	90	62.07	64.98	65.34	90
49	1429.78	1430.30	1430.30	49	91		52.88	53.20	91
50	1372.29	1372.86	1372.86	50	92			42.58	92
51	1316.85	1317.47	1317.47	51	93			33.45	93
52	1263.36	1264.03	1264.03	52	94			25.74	94
53	1211.71	1212.45	1212.46	53	95			19.38	95
54	1161.83	1162.65	1162.66	54	96			14.23	96
55	1113.60	1114.52	1114.55	55	97			10.18	97
56	1066.96	1067.98	1068.03	56	98			7.08	98
57	1021.82	1022.95	1023.04	57	99			4.77	99
58	978.09	979.35	979.49	58	100			3.11	100

AMC00 Male Life Functions (4%)

Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x	Age x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	Age x
17	121013.22		121014.86	17	59	14258.53	14260.18	14260.36	59
18	115879.55	115880.77	115881.13	18	60	13320.96	13322.83	13323.07	60
19	110945.59	110946.76	110947.11	19	61	12424.28	12426.38	12426.69	61
20	106203.56	106204.70	106205.03	20	62	11567.26	11569.63	11570.01	62
21	101646.03	101647.13	101647.45	21	63	10748.78	10751.44	10751.90	63
22	97265.82	97266.89	97267.20	22	64	9967.77	9970.73	9971.28	64
23	93056.05	93057.08	93057.39	23	65	9223.22	9226.50	9227.15	65
24	89010.09	89011.10	89011.39	24	66	8514.21	8517.82	8518.56	66
25	85121.59	85122.57	85122.86	25	67	7839.85	7843.79	7844.64	67
26	81384.45	81385.41	81385.69	26	68	7199.32	7203.60	7204.55	68
27	77792.78	77793.72	77793.99	27	69	6591.87	6596.47	6597.52	69
28	74340.96	74341.87	74342.13	28	70	6016.77	6021.68	6022.82	70
29	71023.58	71024.44	71024.70	29	71	5473.35	5478.53	5479.75	71
30	67835.43	67836.24	67836.49	30	72	4960.98	4966.38	4967.67	72
31	64771.51	64772.26	64772.48	31	73	4479.04	4484.61	4485.94	73
32	61827.01	61827.70	61827.89	32	74	4026.96	4032.62	4033.96	74
33	58997.30	58997.93	58998.09	33	75	3604.16	3609.81	3611.14	75
34	56277.95	56278.53	56278.66	34	76	3210.07	3215.62	3216.88	76
35	53664.69	53665.23	53665.34	35	77	2844.13	2849.44	2850.58	77
36	51153.45	51153.95	51154.04	36	78	2505.65	2510.66	2511.63	78
37	48740.30	48740.77	48740.84	37	79	2193.87	2198.61	2199.39	79
38	46421.48	46421.92	46421.97	38	80	1907.94	1912.56	1913.17	80
39	44193.38	44193.79	44193.82	39	81	1647.13	1651.72	1652.24	81
40	42052.51	42052.91	42052.93	40	82	1410.71	1415.31	1415.80	82
41	39995.55	39995.95	39995.96	41	83	1197.92	1202.49	1202.97	83
42	38019.32	38019.72	38019.72	42	84	1007.84	1012.34	1012.82	84
43	36120.76	36121.17	36121.17	43	85	839.45	843.83	844.31	85
44	34296.94	34297.35	34297.35	44	86	691.61	695.84	696.30	86
45	32545.06	32545.48	32545.48	45	87	563.10	567.14	567.58	87
46	30862.41	30862.85	30862.85	46	88	452.62	456.42	456.84	88
47	29246.43	29246.89	29246.89	47	89	358.78	362.31	362.70	89
48	27694.65	27695.14	27695.14	48	90	280.12	283.35	283.71	90
49	26204.72	26205.24	26205.24	49	91		218.05	218.37	91
50	24774.37	24774.94	24774.94	50	92			165.17	92
51	23401.47	23402.08	23402.08	51	93			122.59	93
52	22083.94	22084.61	22084.61	52	94			89.14	94
53	20819.83	20820.58	20820.58	53	95			63.39	95
54	19607.27	19608.12	19608.13	54	96			44.02	96
55	18444.47	18445.44	18445.47	55	97			29.78	97
56	17329.76	17330.87	17330.93	56	98			19.60	98
57	16261.53	16262.80	16262.89	57	99			12.52	99
58	15238.27	15239.71	15239.85	58	100			7.75	100

AMC00 Male Life Functions (4%)

Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x	Age x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	Age x
17	478.10		479.31	17	59	387.30	388.64	388.82	59
18	475.89	476.71	477.05	18	60	382.24	383.72	383.95	60
19	473.75	474.54	474.88	19	61	376.79	378.43	378.73	61
20	471.68	472.45	472.77	20	62	370.93	372.74	373.11	62
21	469.68	470.42	470.74	21	63	364.64	366.64	367.09	63
22	467.74	468.47	468.77	22	64	357.89	360.09	360.62	64
23	465.87	466.57	466.86	23	65	350.67	353.07	353.70	65
24	464.05	464.73	465.02	24	66	342.95	345.57	346.29	66
25	462.28	462.94	463.22	25	67	334.71	337.55	338.37	67
26	460.56	461.21	461.48	26	68	325.95	329.02	329.93	68
27	458.89	459.52	459.78	27	69	316.66	319.94	320.95	69
28	457.25	457.86	458.12	28	70	306.83	310.32	311.42	70
29	455.66	456.25	456.50	29	71	296.46	300.15	301.32	71
30	454.11	454.67	454.91	30	72	285.56	289.42	290.66	72
31	452.60	453.13	453.34	31	73	274.15	278.16	279.44	73
32	451.12	451.62	451.80	32	74	262.26	266.38	267.67	74
33	449.65	450.12	450.27	33	75	249.92	254.10	255.37	75
34	448.18	448.63	448.76	34	76	237.17	241.36	242.57	76
35	446.72	447.14	447.25	35	77	224.08	228.21	229.31	77
36	445.24	445.65	445.74	36	78	210.67	214.70	215.64	78
37	443.76	444.15	444.22	37	79	196.93	200.88	201.63	79
38	442.25	442.64	442.69	38	80	182.84	186.76	187.35	80
39	440.72	441.10	441.13	39	81	168.47	172.39	172.90	81
40	439.16	439.53	439.55	40	82	153.96	157.90	158.37	82
41	437.54	437.92	437.93	41	83	139.50	143.41	143.88	83
42	435.88	436.26	436.26	42	84	125.24	129.10	129.56	84
43	434.15	434.54	434.54	43	85	111.32	115.08	115.54	85
44	432.35	432.75	432.75	44	86	97.87	101.50	101.94	86
45	430.47	430.88	430.88	45	87	85.02	88.49	88.91	87
46	428.50	428.92	428.92	46	88	72.90	76.17	76.57	88
47	426.42	426.87	426.87	47	89	61.63	64.67	65.04	89
48	424.23	424.70	424.70	48	90	51.30	54.08	54.42	90
49	421.91	422.41	422.41	49	91		44.49	44.80	91
50	419.43	419.98	419.98	50	92			36.23	92
51	416.80	417.39	417.39	51	93			28.74	93
52	413.98	414.62	414.62	52	94			22.32	94
53	410.95	411.66	411.66	53	95			16.94	95
54	407.70	408.49	408.50	54	96			12.54	96
55	404.20	405.08	405.10	55	97			9.04	97
56	400.43	401.41	401.46	56	98			6.33	98
57	396.37	397.46	397.55	57	99			4.29	99
58	392.01	393.21	393.34	58	100			2.81	100

AMC00 Male Life Functions (4%)

Age x	$A_{[x]}$	A_x	$\dot{a}_{[x]}$	\ddot{a}_x	Age x	Age x	$A_{[x]}$	A_x	$\dot{a}_{[x]}$	\ddot{a}_x	Age x
17	0.09315	0.09337	23.578	23.572	17	59	0.41391	0.41483	15.238	15.214	59
18	0.09647	0.09669	23.492	23.486	18	60	0.42728	0.42834	14.891	14.863	60
19	0.09993	0.10014	23.402	23.396	19	61	0.44087	0.44209	14.537	14.506	61
20	0.10352	0.10373	23.309	23.303	20	62	0.45467	0.45607	14.179	14.142	62
21	0.10725	0.10747	23.211	23.206	21	63	0.46866	0.47025	13.815	13.774	63
22	0.11114	0.11135	23.110	23.105	22	64	0.48281	0.48462	13.447	13.400	64
23	0.11517	0.11539	23.005	23.000	23	65	0.49711	0.49916	13.075	13.022	65
24	0.11937	0.11959	22.896	22.891	24	66	0.51154	0.51384	12.700	12.640	66
25	0.12373	0.12395	22.783	22.777	25	67	0.52608	0.52863	12.322	12.256	67
26	0.12826	0.12848	22.665	22.659	26	68	0.54069	0.54352	11.942	11.869	68
27	0.13297	0.13320	22.543	22.537	27	69	0.55536	0.55847	11.561	11.480	69
28	0.13787	0.13809	22.415	22.410	28	70	0.57005	0.57345	11.179	11.090	70
29	0.14296	0.14318	22.283	22.277	29	71	0.58476	0.58843	10.796	10.701	71
30	0.14825	0.14847	22.146	22.140	30	72	0.59945	0.60338	10.414	10.312	72
31	0.15375	0.15396	22.003	21.997	31	73	0.61411	0.61827	10.033	9.925	73
32	0.15946	0.15966	21.854	21.849	32	74	0.62870	0.63306	9.654	9.541	74
33	0.16539	0.16558	21.700	21.695	33	75	0.64322	0.64772	9.276	9.159	75
34	0.17154	0.17172	21.540	21.535	34	76	0.65765	0.66222	8.901	8.782	76
35	0.17792	0.17809	21.374	21.370	35	77	0.67197	0.67653	8.529	8.410	77
36	0.18454	0.18471	21.202	21.198	36	78	0.68613	0.69062	8.161	8.044	78
37	0.19141	0.19157	21.023	21.019	37	79	0.70005	0.70445	7.799	7.684	79
38	0.19853	0.19868	20.838	20.834	38	80	0.71360	0.71800	7.446	7.332	80
39	0.20590	0.20605	20.647	20.643	39	81	0.72672	0.73123	7.105	6.988	81
40	0.21354	0.21369	20.448	20.444	40	82	0.73942	0.74413	6.775	6.652	82
41	0.22145	0.22160	20.242	20.238	41	83	0.75173	0.75668	6.455	6.326	83
42	0.22963	0.22978	20.030	20.026	42	84	0.76365	0.76884	6.145	6.010	84
43	0.23810	0.23826	19.809	19.805	43	85	0.77518	0.78060	5.845	5.704	85
44	0.24685	0.24702	19.582	19.578	44	86	0.78629	0.79195	5.556	5.409	86
45	0.25590	0.25608	19.347	19.342	45	87	0.79698	0.80287	5.279	5.125	87
46	0.26524	0.26543	19.104	19.099	46	88	0.80724	0.81336	5.012	4.853	88
47	0.27488	0.27509	18.853	18.848	47	89	0.81705	0.82340	4.757	4.592	89
48	0.28483	0.28506	18.594	18.589	48	90	0.82642	0.83299	4.513	4.342	90
49	0.29508	0.29533	18.328	18.321	49	91		0.84213		4.105	91
50	0.30564	0.30591	18.053	18.046	50	92		0.85081		3.879	92
51	0.31651	0.31681	17.771	17.763	51	93		0.85905		3.665	93
52	0.32768	0.32801	17.480	17.472	52	94		0.86683		3.462	94
53	0.33915	0.33953	17.182	17.172	53	95		0.87417		3.272	95
54	0.35091	0.35135	16.876	16.865	54	96		0.88106		3.092	96
55	0.36297	0.36347	16.563	16.550	55	97		0.88751		2.925	97
56	0.37530	0.37589	16.242	16.227	56	98		0.89351		2.769	98
57	0.38791	0.38859	15.914	15.897	57	99		0.89904		2.625	99
58	0.40079	0.40158	15.580	15.559	58	100		0.90404		2.495	100

End of Appendix.