Queen Mary University of London

Main Examination period 2020 – May/June – Semester B
Online Alternative Assessments

MTH5103: Complex Variables

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment, you may use books, notes, and the internet. You may use calculators and computers, but you should show your working for any calculations you do. You must not seek or obtain help from anyone else.

At the start of your work, please **copy out and sign** the following declaration:

> I declare that my submission is entirely my own, and I have not sought or obtained help from anyone else.

All work should be **handwritten**.

You have **24 hours** in which to complete and submit this assessment. When you have finished your work, you should upload photographs or scans of your work using the upload tool on the QMplus page for the module. You should also email a copy of your work to maths@qmul.ac.uk with your student number and the module code in the subject line.

One of the photographs you upload should feature the first page of your work together with either yourself or your student ID card.

You are not expected to spend a long time working on this assessment. We expect you to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the assessment period, in case you experience computer problems.

Examiners: M. Shamis, O. Jenkinson

Question 1 [20 marks].

(a) Compute the following number

$$\left|\frac{(2-2i)^2(3i-4)}{(i+i^2+\cdots+i^6)^6}\right|.$$

Justify all of your steps. **[10]**

(b) Find all the solutions $z \in \mathbb{C}$ of the equation

$$z^7 = 4 + 3i.$$

Please write the answer either in Cartesian or in polar form. **[10]**

Question 2 [20 marks].

(a) Using the Root test, or otherwise, determine the radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n} z^n.$$

State the test that you are using and justify all of your steps. **[7]**

(b) Using the Ratio Test, or otherwise, determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}(z-2i)^n.$$

State the test that you are using and justify all of your steps. **[10]**

(c) Does the series in part (b) converge for $z = 2i - 2$? Please justify your answer using an appropriate test for convergence or divergence. **[3]**

Question 3 [20 marks].

(a) Determine the residue of the function

$$f(z) = z^2 \sin \frac{1}{z}$$

at the point $z = 0$. **[10]**

(b) Find the coefficients a_n and b_n of the Laurent series

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} b_n (z - z_0)^{-n}$$

for the function

$$f(z) = z^2 \sin \frac{1}{z}$$

from (a) on the set $\{z \in \mathbb{C} : |z| > 0\}$ (for $z_0 = 0$). Where is this series absolutely convergent? **[10]**

Question 4 [20 marks].

(a) Find all the analytic functions f for which, for all $z \in \mathbb{C}$

$$\operatorname{Im} z = \operatorname{Re} f(z).$$

Please provide full explanation to the answer and justify all your steps. **[10]**

(b) Find all singularities of the function

$$f(z) = \frac{e^{-\frac{1}{z}}}{z^2 - \frac{\pi^2}{4}},$$

and determine the nature of each of these singularities (e.g. removable singularity, simple pole, double pole, essential singularity etc.). Justify all your steps. **[10]**

Question 5 [20 marks].

(a) Using the Residue Theorem, or otherwise, compute

$$\int_C \frac{\cos z}{(z-\pi/2)(z^2+25)}\mathrm{dz},$$

where C is the positively oriented circle of radius 3 centred at $z = 0$. State all the theorems and/or propositions that you use during the computation. Justify all your steps. [**10**]

(b) Using the Residue Theorem, or otherwise, compute

$$\int_C \frac{\sin z}{z(z-1)^2}\mathrm{dz},$$

where C is the positively oriented circle of radius 2 centred at $z = 0$. State all the theorems and/or propositions that you use during the computation. Justify all your steps. [**10**]

End of Paper.