

MTH5102: Calculus III

**Duration: 2 hours** 

Date and time: 16 May 2016, 10:00–12:00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately. It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it shall be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

**Examiner(s): L. Lacasa** 

Page 2 MTH5102 (2016)

**Question 1 (10 marks).** Let  $\mathcal{C}$  be the curve in  $\mathbf{R}^3$  whose parametric equation reads  $\mathbf{r}(t) = (\frac{t}{2}, \frac{t^2}{2}, \frac{2t^{3/2}}{3})$ . Consider the points A = (0,0,0), B = (1/2,1/2,2/3) and C = (1,2,8/3).

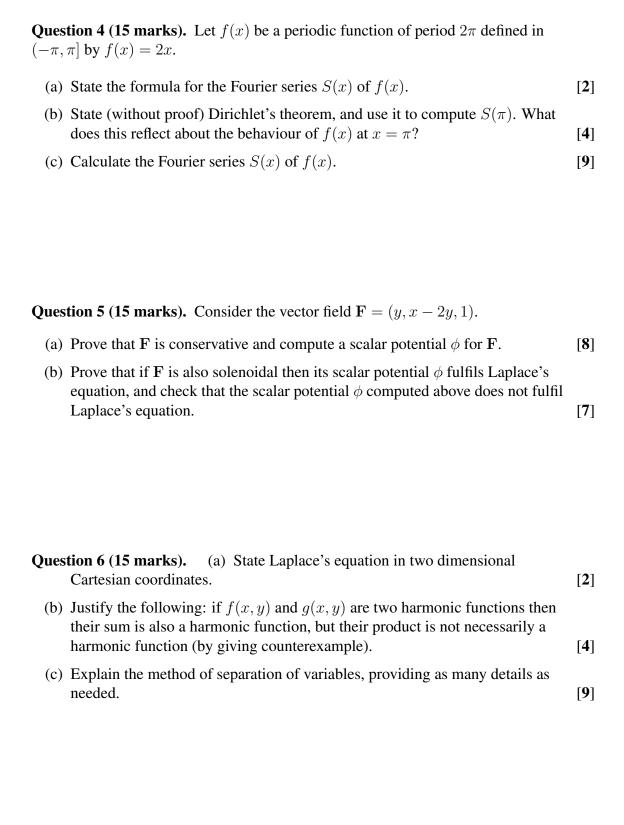
- (a) Show that A, B are in the curve and determine whether or not C is on the curve (justify your answer). [2]
- (b) Compute the arc of length of  $\mathcal{C}$  between the points A and B. [8]

Question 2 (25 marks). Let C be an ellipse in the XY plane described in implicit form by  $x^2 + 4y^2 = 1$ , and let  $\mathbf{F} = x\mathbf{i} + \mathbf{j}$  be a vector field.

- (a) Find a parametric equation for C. [3]
- (b) Compute the line integral of  $\mathbf{F}$  along  $\mathcal{C}$  (travelled anticlockwise) from A=(1,0) to B=(-1,0). [7]
- (c) Make a sketch of **F** and use the sketch to deduce the value of  $\nabla \times \mathbf{F}$ . [5]
- (d) State Stokes's theorem and use this along with previous part of the question to deduce the line integral of  $\mathbf{F}$  along  $\mathcal{C}$  (travelled anticlockwise) from B to A. Give as many details as needed. [10]

**Question 3 (20 marks).** Let S be a sphere of radius a centred at the origin and let  $F = z\mathbf{k}$  be a vector field.

- (a) Give a parametric equation for S along with the range of the parameters. [5]
- (b) State the divergence theorem and use it, explaining why it can be applied, to compute the flux of F across S. [5]
- (c) Express F and dS in spherical coordinates. [6]
- (d) Using the spherical coordinate system, compute the flux of F across S. [4]



Page 3

MTH5102 (2016)