Queen Mary
University of London

Main Examination period 2020 – January – Semester A

MTH4107 / MTH4207: Introduction to Probability

Duration: 2 hours

Student number | | | | | | | | | | **Desk number** | | | |

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Write your solutions in the spaces provided in this exam paper. If you need more paper, ask an invigilator for an additional booklet and attach it to this paper at the end of the exam.

You should attempt ALL questions. Marks available are shown next to the questions.

Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough work in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any unauthorised notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: R. J. Harris, W. Just

This page is for marking purposes only.

Do not write on it.

Question	Mark	Comments
1	/ 20	
2	/ 20	
3	/ 20	
4	/ 20	
5	/ 20	
Total		

In this exam paper, $\mathbb{P}$ denotes probability, $\mathbb{E}$ denotes expectation, and Var denotes variance. You should show your working and explain the steps in your solutions, except where you are simply asked to "State" or "Write down" a result. Your final answers should be simplified as far as possible.

Question 1 [20 marks]. An unknown person has broken into the office of Dr Harris and added extra "dots" to her giant die. One day she notices that the die now has two faces with 6's, three faces with 5's, and one face with a 3. She rolls this modified die twice in her next lecture.

(a) Write down the sample space for this experiment. [**5**]

(b) State the following events as subsets of the sample space: [**4**]

A: “The first roll is not a prime number”;
B: “The sum of the two rolls is less than nine”.

(c) Despite the modification, each of the six faces is still equally likely to appear. Compute the probabilities of the events A and B from part (b). [**5**]

(d) Are the events A and B mutually exclusive? Are they independent? Justify your answers. [**6**]

Question 2 [20 marks]. A group of six Maths students and four Geography students always eat lunch together at The Curve.

(a) One day, three students are chosen at random from this group to take part in the Nutrition Satisfaction Survey. Compute the probability that the three students selected all study the same subject. [**8**]

(b) Given that the three students selected in the survey of part (a) all study the same subject, find the probability that they are Maths students. [**5**]

(c) Another day, a single student from the same group of ten friends is chosen at random to fill in a Beverage Feedback Questionnaire. Seven of the students in the group like coffee. Letting C be the event that the student filling in the questionnaire likes coffee and M be the event that the student filling in the questionnaire is a Maths student, show that $\mathbb{P}(C|M) \geq 1/2$. [**7**]

Question 3 [20 marks].

(a) A biased coin has probability 2/3 of coming up heads. You toss the coin six times and record the total number of heads X.

(i) State the name for the distribution of X, and specify its parameters. [**2**]

(ii) Using known results for this distribution, or otherwise, obtain $\mathbb{E}(X)$ and $\text{Var}(X)$. [**4**]

(b) A geometrically distributed random variable Y has probability mass function

$$\mathbb{P}(Y = k) = \frac{1}{3}\left(\frac{2}{3}\right)^{k-1} \quad \text{for } k \in \{1, 2, 3, \ldots\}.$$

Show that $\mathbb{P}(Y > k) = (2/3)^k$, and hence derive an expression for $\mathbb{P}(Y > k+2 | Y > k)$. **[7]**

(c) The number of lecturers L that you encounter per day when walking around the campus is a Poisson random variable with parameter $\lambda = 2$.

(i) Write down the probability mass function $\mathbb{P}(L = k)$ for encountering k lecturers in a day. [**3**]

(ii) Calculate the probability that you encounter more than the expected number of lecturers in a day. [Your final answer should be written as a closed-form expression involving powers of e, not an infinite sum.] [**4**]

Question 4 [20 marks]. Two integer-valued random variables, X and Y, have joint probability mass function given by the following table.

$Y \backslash X$	0	1	2
0	$6a$	$3a$	$2a$
1	0	$3a$	$2a$
2	0	0	$2a$

(a) Determine the value of a, and hence rewrite the probability mass function with the table entries as fractions. [**3**]

(b) Find the marginal probability mass function of X. Hence determine the value of $\mathbb{E}(X)$. [**5**]

(c) Find the value of $\mathrm{Var}(3X+2)$. [**5**]

(d) Compute the value of the covariance of X and Y. [You may use without proof that $\mathbb{E}(Y) = 1/2$.] [**5**]

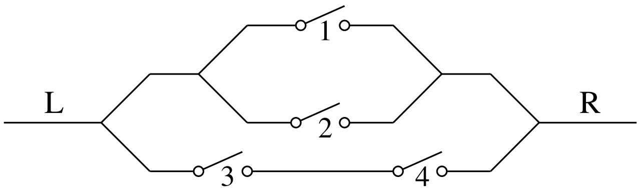
(e) Explain in words the meaning of the covariance being positive. [**2**]

Question 5 [20 marks].

(a) State **Kolmogorov's Axioms** of probability. [**6**]

(b) Using the Inclusion-Exclusion Principle or otherwise, show that if events A and B are independent, then A^c and B^c are also independent. [**7**]

(c) The figure below shows part of the lighting circuit in the new Maths Building. Electric current can flow from L to R either through the upper branch or the lower branch. The upper branch contains two switches, 1 and 2, connected in parallel; current can flow through that branch if and only if at least one of 1 or 2 is closed. The lower branch contains two switches, 3 and 4, connected in series; current can flow through that branch if and only if both 3 and 4 are closed. The events that the switches are closed are mutually independent and each switch is closed with probability p. Obtain, in terms of p, the probability that current can flow from L to R. [**7**]



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