Queen Mary
University of London

Main examination period 2017

MTH4105 Introduction to Mathematical Computing

Duration: 2 hours

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators may be used in this examination.

Answer each question in the appropriate subsection headed *Answers* in the Exam answer template provided in QMplus. You must use Maple to perform all calculations. Do not delete any relevant input or output; you will score marks only for what is visible in the document you submit. There may be more than one correct solution to each question; any working solution will be accepted provided it satisfies the requirements of the question.

This exam is open book. You may access any information you want, but must work entirely by yourself. You may not communicate, nor attempt to communicate, with anyone else, nor solicit assistance in any way. Use of mobile phones and similar communication devices is prohibited. Please be aware that details of all internet activity on your computer may be logged. You may do rough work on your own paper, which will not be collected by the invigilators. A mobile phone that causes a disruption in the exam is an assessment offence.

Exam papers must not be removed from the examination room.

Examiners: F. J. Wright, V. Nicosia

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Question 1 [12 marks]

a) Assign the expression $x^2 + \frac{2}{x^2}$ to the variable y. You must use the variable y and not the expression $x^2 + \frac{2}{x^2}$ in the rest of this question. **[2 marks]**

b) Plot the graph of y against x for $-3 \le x \le 3$, $0 \le y \le 10$, labelling the horizontal and vertical axes with the variable names x and y respectively. **[6 marks]**

c) Compute all the **exact** values of x for which $\frac{d}{dx}\, y = 0$ by solving the equation. **[2 marks]**

d) Evaluate y at $x = 2^{1/4}$. **[2 marks]**

Question 2 [12 marks]

a) Assign to the variable S the set of integers from 1000 to 1100 inclusive, without inputting all the elements explicitly. **[2 marks]**

b) Assign to the variable P the subset of S obtained by evaluating an expression that selects the elements of S that are prime; mathematically $P = \{s \in S : s \textit{ prime}\}$. **[2 marks]**

c) Compute the cardinality of P. **[2 marks]**

d) Assign to the variable Q the set of integers of the form $p + 2$, where p is an element of P, obtained by evaluating an expression that constructs this set from P; mathematically $Q = \{p + 2 : p \in P\}$. **[3 marks]**

e) Show that at least one element of Q is divisible by 7 by entering **a single expression** that evaluates to *true* or *false* as appropriate. **[3 marks]**

Question 3 [12 marks]

Write and execute a program that computes and assigns to the variable S the set of all positive integer divisors of a positive integer n as follows. Your program should all be in one execution group.

1) Assign the value 100 to the variable n. **[2 marks]**

2) Assign the empty sequence to the variable S. **[2 marks]**

3) Then use a **do** statement (loop) to run through all the integers m such that $1 \le m \le n$ and within the **do** statement: **[2 marks]**

    a) test whether each value of m divides n **[2 marks]**

    b) and if so include it in the sequence S. **[2 marks]**

4) Finally, make the sequence S into a set and assign it back to the variable S. **[2 marks]**

Display the assignment to n and the final assignment of the set of all divisors of n to S, but nothing else.

Question 4 [12 marks]

a) Define f to be a formula-based function such that $f(n) = \frac{n\cdot(n+1)}{2}$. **[3 marks]**

b) Define g to be a table-based function such that
$g(0) = 0$, $g(1) = 1$, $g(2) = 3$, $g(3) = 6$, $g(4) = 10$. **[3 marks]**

c) Let $A = \{0, 1, 2, 3, 4\}$ be the domain of f. Assign to the variable B the range of f computed using the domain, A. **[3 marks]**

d) If f and g are both regarded as functions with domain A and codomain B, show that $f = g$ by entering and evaluating a **single expression** that evaluates to *true* or *false* as appropriate. **[3 marks]**

Question 5 [14 marks]

Let $A = \{1, 2, 3, 4, 5\}$.

a) Assign to the variable S the set of all 2-element subsets of A, computed using A. **[5 marks]**

b) Define $\&R$ to be a relation on A such that $x \,\&R\, y$ evaluates to *true* if $\{x, y\}$ is an element of S and to *false* otherwise. **[3 marks]**

c) Evaluate $2 \,\&R\, 4$, $4 \,\&R\, 2$. **[2 marks]**

d) Explain **in a text region**, with all mathematical notation in math mode, why $\&R$ must be symmetric. **[4 marks]**

Question 6 [14 marks]

The solutions of the equations $x^2 + y^2 = 1$ and $x^2 - y^2 = 1$ form a circle and a hyperbola respectively in the (x, y)-plane.

a) Plot these two **implicitly defined curves** together on the same axes, using for both axes the same range $[-2, 2]$ and the **same scaling**. **[8 marks]**

b) The circle and hyperbola defined above intersect at four points in the plane. Solve the pair of simultaneous equations to find **exact explicit** expressions for the x and y coordinates of the four intersections. **[6 marks]**

Question 7 [12 marks]

Write a **procedure** called *sum_of_two_primes* that takes one parameter, *n*, which you can assume to be an even positive integer greater than 2, and returns an expression sequence consisting of two primes that sum to *n*, as follows.

1) Write the procedure skeleton and include a brief Maple comment to explain its purpose. **[2 marks]**
2) Declare local variables as appropriate and assign the value 0 to a local variable *p*. **[2 marks]**
3) Then use a **do** statement (loop) that repeats at most *n* times and within the **do** statement: **[2 marks]**
   a) assign to *p* the next prime number after the current value of *p*; **[2 marks]**
   b) if $n-p$ is prime then return the expression sequence $p, n-p$. **[4 marks]**

Your procedure should behave as follows:

```
> sum_of_two_primes(4)
                                  2, 2
> sum_of_two_primes(60)
                                  7, 53
```

Question 8 [12 marks]

a) Set the Maple interface to represent the imaginary unit as the lower case letter i. **[2 marks]**

b) Assign the complex variable expression $\mathrm{i}\,w$ to the variable z and then assign the complex number $2+3\mathrm{i}$ to the variable w. **[2 marks]**

c) By using a single expression that evaluates to *true* or *false* as appropriate, show that z and w have the same modulus. **[2 marks]**

d) Compute the difference between the arguments of z and w in the simplest possible exact form. **[2 marks]**

e) Plot the complex numbers represented by z and w as points on an Argand diagram joined to the origin by straight lines. Ensure that this plot is geometrically correct. **[4 marks]**

End of Paper