Queen Mary University of London

School of Mathematical Sciences

BSc Examination by course unit

MTH4105 Introduction to Mathematical Computing

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Friday 23 May 2014, 10:00–12:00 **Duration: 2 hours**

If you have not already done so, save this file to your My Documents folder. Re-save it frequently.

Before you begin this exam, enter in the table below your **name** (as registered with Queen Mary) and **student number** (the one that consists of 9 digits).

First forename (or given name):	
Surname (or family name):	
Student number:	
Marks – for examiner's use only:	`+`(_Q‖(1..8))

You are not permitted to expand the collapsed section below until instructed to begin the exam by an invigilator.

Answer ALL questions

Answer each question in the appropriate subsection headed *Answers*. You must use Maple to perform all calculations. Do not delete any relevant input or output; you will score marks only for what is visible in the document you submit. There may be more than one correct solution to each question; any working solution will be accepted provided it satisfies the requirements of the question.

This exam is open book. You may access any information you want, but must work entirely by yourself. You may not communicate, nor attempt to communicate, with anyone else, nor solicit assistance in any way. Please be aware that details of all internet activity on your computer may be logged. You may do rough work on your own paper, which will not be collected by the invigilators.

You may use any form of input and output but **do not use the right mouse button or use the mouse to copy Maple output** because this may cause Maple to stop responding. To refer to Maple output in subsequent input, use its label via the *Insert* menu or *Ctrl*+L. You are recommended to restart the Maple server before answering each main question by executing the *restart* commands provided in the *Answers* subsections. **Insert additional execution groups as necessary. Note that you are not required to use two-dimensional input and you can use functional equivalents to the templates in the palettes if you wish.**

Before you leave the exam room you must upload this .mw file to QMplus using the upload link in QMplus immediately below the download link **and also email it to f.j.wright@qmul.ac.uk** from your QMUL email account as backup.

Examiners: F. J. Wright, W. Just

Click this triangle when you are told to begin the exam

Question 1 [12 marks]

a) Assign to the variable *y* the trigonometric cosine function applied to the square of the variable *x*. **[2 marks]**

b) Use the variable *y* at least once to assign to the variables *y1* and *y2* respectively the first and second derivatives of *y* with respect to the variable *x*. **[3 marks]**

c) Use the variables *y*, *y1*, *y2* to plot together on the same axes the graphs of *y*, *y1*, *y2* as functions of *x* for $0 \leq x \leq \frac{3}{4}\pi$ with a vertical range from -5 to 15 and a legend that identifies the graphs clearly with the labels *y*, *y1*, *y2* (and not the values of these variables). **[4 marks]**

d) Let *x0* be the smallest positive value of *x* at which *y* is a local minimum (i.e. its first derivative *y1* is zero and its second derivative *y2* is positive). Using your plot as guidance, find an *x*-range that contains *x0* and no other value of *x* at which *y* is stationary and assign this range to the variable *x0range*. Use *x0range* to compute a numerical approximation to *x0* and assign it to the variable *x0*. As a check, compute the value of the variable *y1* at this value of *x*. **[3 marks]**

Answers

Before starting this question, execute this command:

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> restart
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a)

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b)

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c)

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d)

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Question 2 [12 marks]

Use the Maple *seq* function or $ operator once to construct each of the sets (a), (b), (c) below and twice to construct the set (d). Use ranges where possible and do not write any output set explicitly.

a) $\{0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$ **[3 marks]**

b) $\{x, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^{10}\}$ **[3 marks]**

c) $\{a^a, b^b, c^c, d^d, e^e\}$ **[3 marks]**

d) $\{\{5\}, \{4, 5\}, \{3, 4, 5\}, \{2, 3, 4, 5\}, \{1, 2, 3, 4, 5\}\}$ **[3 marks]**

Answers

Before starting this question, execute this command:

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> restart
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a)

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b)

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c)

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d)

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Question 3 [12 marks]

a) Assign the set $\{0, 1, 2, 3, 4\}$ to the variable A and the set $\{1, 2, 3, 4, 5\}$ to the variable B. **Use only the variables *A* and *B* in the following. [2 marks]**

b) Enter and execute a single expression in the variables A and B that evaluates to *true* if A is not a subset of B and B is not a subset of A, and to *false* otherwise. **[4 marks]**

c) Enter and execute a single expression in the variables A and B that evaluates to the set of elements that are members of one but not both of the sets A and B. **[3 marks]**

d) Enter and execute a single expression in the variables A and B that evaluates to the smallest set that contains both A and B. **[3 marks]**

Answers

Before starting this question, execute this command:

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a)
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b)
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c)
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d)
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Marks – for examiner's use only

Question 4 [12 marks]

a) Define the continuous formula-based function $f:\mathbb{R}\to\mathbb{R}$ such that $x \mapsto 8(x^3+x^2+x)$. **[3 marks]**

b) Let $A = \left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$. Define the discrete table-based function $g: A\to\mathbb{R}$ such that $-1 \mapsto -8, -\frac{1}{2} \mapsto -3, 0 \mapsto 0, \frac{1}{2} \mapsto 7, 1 \mapsto 24$. **[3 marks]**

c) Enter and execute a single expression that evaluates to *true* if the function *f* restricted to the domain *A* is equal to the function *g*. **[3 marks]**

d) Define the function *G* to be the inverse of the function *g*. (You may assume that *g* is invertible.) **[3 marks]**

Answers

Before starting this question, execute this command:

> *restart*

a)
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b)
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c)
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d)
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Question 5 [12 marks]

Suppose that A evaluates to the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and P evaluates to the set of sets $\{\{1, 3\}, \{2, 4, 5\}, \{6, 9\}, \{7, 8\}\}$. Use this example to test the following.

a) Define the neutral operator $\&R$ such that $a \&R\, b$ evaluates to *true* if both operands a and b are elements of the same element of P, and *false* otherwise. Test it by evaluating $1 \&R\, 2$, $2 \&R\, 3$, $3 \&R\, 4$, $4 \&R\, 5$. **[6 marks]**

b) Define the function Q such that $Q(b) = \{a \in A : a \&R\, b\}$ and compute the result of mapping Q over A. **[6 marks]**

Answers

Before starting this question, execute this command:

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a)

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b)

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Marks – for examiner's use only

Question 6 [12 marks]

A twin prime pair is a pair of prime numbers that differ by 2. One way to compute twin prime pairs is to initialize a variable p to the smallest prime, 2, and then successively increment p to the next prime in a loop, within which you also check whether p is the smaller of a twin prime pair and if so print the pair. Enter and execute a program that displays the smallest 10 twin prime pairs. Do not display any other output. **[12 marks]**

Answers

Before starting this question, execute this command:

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> restart
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Now save this document.

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Question 7 [12 marks]

a) Write a loop with suitable initialization that computes and displays the product $n \times (n - m) \times (n - 2\,m) \times \cdots \times \ell$ where $m \ge \ell > 0$, i.e. ℓ is as small as possible whilst remaining positive. Do not display any other output. Assign 10 to n and 3 to m, and execute your code; it should display the result 280. **[8 marks]**

b) Write a procedure (or function) called *MultiFac* that takes two arguments, n and m, both of which you can assume to be positive integers, and returns the value of the product $n \times (n - m) \times (n - 2m) \times \cdots \times \ell$ where $m \geq \ell > 0$. Declare local variables as appropriate to avoid any warnings from Maple. Evaluate $MultiFac(10, 3)$; it should display the result 280. **[4 marks]**

Answers

Before starting this question, execute this command:

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> restart
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a)

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b)

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Question 8 [16 marks]

In this question, the symbol i represents the imaginary unit.

a) Compute the four solutions of the complex equation $z^4 - 1 = \mathrm{i}$ exactly and assign them (in an appropriate data structure, such as an expression sequence) to the variable *solutions*. Then use the variable *solutions* to plot the solutions as four distinct points on an Argand diagram. **[8 marks]**

b) Plot on an Argand diagram the curve defined by the equation $\Re(z^2) = 1$ for $|\Re(z)| \leq 4$, $|\Im(z)| \leq 4$, where $\Re(z)$, $\Im(z)$ represent respectively the real and imaginary parts of the complex variable z. You can do this by substituting $z = x + \mathrm{i}y$, where the variables x and y are real, and then plotting the curve defined implicitly in the real x, y plane. **[8 marks]**

Answers

Before starting this question, execute this command:

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> restart
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a)

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b)

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Now save this document.

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End of Paper