MTH4104 - INTRODUCTION TO ALGEBRA - 2021/22

YOU CAN PREVIEW THIS QUIZ, BUT IF THIS WERE A REAL ATTEMPT, YOU WOULD BE BLOCKED BECAUSE:

This quiz is not currently available

QUESTION 1

Not yet answered Marked out of 10.00

Let m be a positive integer. Define the set $R = \{0, 1, 2, \ldots, m-1\}$. Define new operations $\oplus$ and $\odot$ on R as follows: for elements $a, b \in R$,

$$a \oplus b := (a + b) \bmod m$$
$$a \odot b := (ab) \bmod m$$

where mod is the binary remainder operation (notes section 2.1). You may assume that R with the operations $\oplus$ and $\odot$ is a ring.

i. What is the difference between the rings R and $\mathbb{Z}_m$? [5 marks]
ii. Explain how the rings R and $\mathbb{Z}_m$ are similar. [5 marks]

Maximum file size: Unlimited, maximum number of files: 1

Files

Drag and drop files here or click to upload

Accepted file types
Image (JPEG) .jpeg
PDF document .pdf

QUESTION 2

Not yet answered Marked out of 10.00

Solve the equation $[8]_{14}\,X + [1]_{14} = [3]_{14}\,X + [12]_{14}$ for $X \in \mathbb{Z}_{14}$. Write your answer as $X = [x]_{14}$ where $0 \le x < 14$. What is x?

Answer:

QUESTION 3

Not yet answered Marked out of 10.00

Let X be a set. Let P be a set of subsets of X such that:

- if A and B are distinct elements of P, then $A \cap B = \varnothing$;
- the union of all sets $A \in P$ is X.

Note that these are clauses (b) and (c) of the definition of a partition (Definition 1.5).

Now define a relation R on the set X by $R = \{ (x, y) : x \in A \text{ and } y \in A \text{ for some } A \in P \}$, as in Theorem 1.7(b). Which of the following is true?

Select one:

- a. R must be an equivalence relation, and $\{ [x]_R : x \in X \}$ must equal P.
- b. R must be reflexive and transitive but might not be symmetric.
- c. R must be symmetric and transitive but might not be reflexive.
- d. R must be an equivalence relation, but $\{ [x]_R : x \in X \}$ might not be equal to P.
- e. R must be reflexive and symmetric but might not be transitive.

QUESTION 4

Not yet answered Marked out of 4.00

Let R be a ring. Let p and q be elements of $R[x]$ such that $\deg(p) = 1$ and $\deg(q) = 2$. True or false: p and q may be equal as elements of $R[x]$.

Select one:

- True
- False

QUESTION 5

Not yet answered Marked out of 10.00

Below are two sets of real numbers. Exactly one of these sets is a ring, with the usual addition and multiplication operations for real numbers. Select the one which is a ring. [4 marks]

- $\{a/2^n : a \in \mathbb{Z}, n \in \mathbb{N}\}$
- $\{a/2 : a \in \mathbb{Z}\}$

Let R be the ring above. True or false: [2 marks each]

R is a ring with identity.

- True
- False

R is a skewfield.

- True
- False

R is a commutative ring.

- True
- False

QUESTION 6

Not yet answered Marked out of 12.00

Let G be the interval $(1/3, \infty)$. Let ¤ be the operation on G such that, for all $x, y \in G$,

$x ¤ y = 6xy - 2(x+y) + 1$.

i. Write down the identity element e for $(G, ¤)$. You need not write a proof of the identity law. [4 marks]
ii. Prove the inverse law for $(G, ¤)$. [8 marks]

Maximum file size: Unlimited, maximum number of files: 1

Files

Drag and drop files here or click to upload

Accepted file types

Image (JPEG) .jpeg

PDF document .pdf

QUESTION 7

Not yet answered Marked out of 10.00

Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 6 & 3 & 4 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 6 & 1 & 2 & 4 \end{pmatrix}$ be permutations in S_6, written in two-line notation.

What is f in cycle notation? Enter single spaces between the numbers in each cycle. Do not type spaces anywhere else in your answer. [4 marks]

Let $h = f^{-1} \circ g$. What is the second line of h in two-line notation? Enter it as a list of numbers separated by single spaces. [6 marks]

QUESTION 8

Not yet answered Marked out of 10.00

Let $(G, \lozenge)$ be a group and $x \in G$. Suppose H is a subgroup of G that contains x. Which of the following must H also contain? [5 marks]

- ☐ All elements $x \lozenge y$ for $y \in G$
- ☐ x^*, the inverse of x
- ☐ The identity element e of G
- ☐ All "powers" $x \lozenge x$, $x \lozenge x \lozenge x$, ...

Enter the smallest subgroup of $\mathbb{Z}_{13}^{\times}$ containing the element $[9]_{13}$, as a set. Write each congruence class in the form $[b]_{13}$ where $0 \leq b < 13$. You don't have to type out the brackets and subscript "$_{13}$". [5 marks]

QUESTION 9

Not yet answered Marked out of 10.00

Let R be a ring. True or false: the product of two nonzero elements of R must be nonzero. [3 marks]

- True
- False

Let $p = ax^2 + bx + c$ and $q = dx^2 + ex + f$ be two elements of $R[x]$. What is the coefficient of x^4 in the product pq? [3 marks]

Assume a and d are nonzero. If you are given no further information, what can you conclude about the degree of pq? [4 marks]

- The degree of pq can be any integer from 0 to 4, or undefined.
- The degree of pq can be any integer at all, or undefined.
- The degree of pq can be any integer greater than or equal to 4.
- The degree of pq is 4.
- The degree of pq is either 3 or 4.

QUESTION 10

Not yet answered Marked out of 10.00

Let S be the following relation on $\mathbb{C}\setminus\{0\}$:

$$S = \{(x, y) \in (\mathbb{C}\setminus\{0\})^2 : |y/x| = 1\}.$$

Prove that S is an equivalence relation.

Recall that the modulus of a complex number $z = a+bi$ is defined as $|z| = \sqrt{a^2 + b^2}$. In your answer you may use properties of the modulus function without proving them.

Maximum file size: Unlimited, maximum number of files: 1

Files

Drag and drop files here or click to upload

Accepted file types

Image (JPEG) .jpeg

PDF document .pdf

QUESTION 11

Not yet answered Marked out of 4.00

Let $X = \{1, 2, 3, 4, 5, 6\}$. Which of the following is a partition of X?

- ○ $\{1, 3, 5\}$
- ○ $(1\ 2)(3\ 4)(5\ 6)$
- ○ $\{(1, 2), (3, 4), (5, 6)\}$
- ○ $\{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$