QUESTION 1

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Below is a proposition about rings, and a short proof in which the use of the axioms and other properties of rings is not made explicit.

Proposition. Let R be a ring with identity. Suppose that a and b are nonzero elements of R such that $ab = 0$. Then a does not have a multiplicative inverse.

Proof. Suppose that a had a multiplicative inverse. The given equation implies that $a^{-1}ab = 0$. But also $a^{-1}ab = b$, contradicting the fact that $b \neq 0$.

Which property is used to prove $a^{-1}ab = 0$? [ ]

Which two properties are used to prove $a^{-1}ab = b$? Put them in the order you would use them when simplifying $a^{-1}ab$ to b. First [ ], then [ ]

One more property is needed in this proof, but the notation makes it ambiguous exactly where it is used. Which property is this?

[ ]

Please look at the lecture notes to remind yourself e.g. what "Proposition 3.13" is.

identity law for ·	distributive law	definition of inverse for ·	Proposition 3.13	definition of inverse for +
associative law for +	commutative law for +	identity law for +	associative law for ·	cancellation property

QUESTION 2

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Let $f = (1\ 3\ 4\ 6)(2\ 8\ 9\ 5)$ and $g = (1\ 4)(2\ 9\ 5)(6\ 8)$ be permutations in S_9, written in cycle notation.

What is the second line of f in two-line notation? Enter it as a list of numbers separated by single spaces. [ ]

Let $h = f \circ g^{-1}$. What is h in cycle notation? Enter single spaces between the numbers in each cycle. Do not type spaces anywhere else in your answer. [ ]

QUESTION 3

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Let a be a real number. Let S be the relation on $\mathbb{R}[x]$ defined so that $(f, g) \in S$ if and only if $x - a$ is a factor of $g - f$. You may assume S is an equivalence relation. Which of the following are true?

Select one or more:

- ☐ a. S has exactly a equivalence classes.
- ☐ b. If q is the quotient and r the remainder when a polynomial $f \in \mathbb{R}[x]$ is divided by $x - a$, then $(f, q) \in S$.
- ☐ c. Every equivalence class of S contains exactly one constant polynomial.
- ☐ d. Every equivalence class of S contains exactly one polynomial of degree at most 1.
- ☐ e. The equivalence class $[0]_S$ is the set of all polynomials in $\mathbb{R}[x]$ of which a is a root.

QUESTION 4

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Carry out Euclid's algorithm to find $d = \gcd(23, 17)$. In the notation of the lecture notes, $b_0 = 23$ and $b_1 = 17$. Enter the first two remainders, b_2 and b_3, produced by the algorithm.

$b_2 =$ [ ]

$b_3 =$ [ ]

Now carry out the extension to Euclid's algorithm. In the course of the algorithm, you will express d as an integer linear combination of each pair of successive remainders. Enter the integer coefficients that arise for the last three steps.

$d =$ [ ] $\cdot b_2 +$ [ ] $\cdot b_3$

$d =$ [ ] $\cdot b_1 +$ [ ] $\cdot b_2$

$d =$ [ ] $\cdot b_0 +$ [ ] $\cdot b_1$. (This line is the final output of the algorithm. The last number you entered should have absolute value less than 12.)

QUESTION 5

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Let $R = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$. You may assume that R is a ring with the usual matrix addition and multiplication operations. True or false:

R is a commutative ring. [ ]

R is a ring with identity. [ ]

R is a skewfield. [ ]

QUESTION 6

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Let $A_S = \{ S \cup T : T \in \mathrm{P}(\{7, 8, 9\}) \}$. Prove that $P = \{ A_S : S \in \mathrm{P}(\{1, \ldots, 6\}) \}$ is a partition of $\mathrm{P}(\{1, \ldots, 9\})$. Here P is the notation for power set.

Please upload your proof as a single PDF file relating to only this question.

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QUESTION 7

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Let $X = \{1, 2, 3, 4\}$. Let $R = \{ (1, 2), (2, 2), (3, 1), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (a, b) \}$, where a and b are elements of X. If R is a transitive relation on X, what is the pair (a, b)?

Answer:

QUESTION 8

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Fill in the blanks in the table below so that it becomes the Cayley table of a group with elements {a, b, c, d}.

	a	b	c	d
a				c
b			b	
c				
d				

QUESTION 9

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Enter the multiplicative inverse of the element $[22]_{44}\, x + [25]_{44}$ in the ring $\mathbb{Z}_{44}[x]$.

When you enter your answer, make sure that all congruence classes are in the form $[a]_{44}$ with $0 \le a < 44$. You don't have to type the brackets. For example, if your answer is $[2]_{44}\, x^2 + [3]_{44}$, you can type 2x^2+3.

Answer:

QUESTION 10

Not yet answered Marked out of 10.00 Flag question

Let $G = \{ [4k]_{20} : k \in \mathbb{Z} \}$. Prove that G with the operation of addition is a subgroup of $\mathbb{Z}_{20}$.

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