Queen Mary
University of London

Main Examination period 2018

MTH4103/MTH4203 : Geometry I

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: C. Busuioc and O. Jenkinson

Question 1. Let A and B be the points in 3-space with position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix}$ respectively. Determine:

(a) The length of the vector $2\mathbf{a} + \mathbf{b}$; **[4]**

(b) The position vector of the point C for which $OACB$ is a parallelogram; **[3]**

(c) The position vector for the point of intersection of the two diagonals of $OACB$; **[3]**

(d) $\mathbf{a} \cdot \mathbf{b}$; **[3]**

(e) Any non-zero vector that is orthogonal to $\mathbf{a}$; **[3]**

(f) A real number λ such that $\mathbf{a} + \lambda\mathbf{b}$ is a unit vector. **[4]**

Question 2.

(a) What is meant by a **linear equation** in variables $x_1, \ldots, x_n$? **[2]**

(b) What does it mean to say that a linear equation is **degenerate**? **[2]**

(c) What is meant by a **system of linear equations**? **[2]**

(d) What does it mean to say that a system of linear equations is in **echelon form**? **[5]**

(e) Use the method of back substitution to find all solutions to the following system of linear equations in variables x, y, z: **[5]**

$$\left.\begin{aligned} x - 4y + 3z &= 2 \\ 3y - 2z &= -5 \\ 2z &= 8 \end{aligned}\right\}$$

(f) State precisely what your answer to part (e) means regarding the intersection of a specific collection of planes in 3-space. **[4]**

Question 3.

(a) Describe, in detail, the geometric object represented by the Cartesian equation $3x - 5y + 2z = 0$. **[3]**

(b) Describe, in detail, the geometric object represented by the parametric equations **[3]**

$$\left.\begin{aligned} x &= 1 + 2\lambda \\ y &= 2 + \lambda \\ z &= 3 - \lambda \end{aligned}\right\}, \qquad \lambda \in \mathbb{R}.$$

(c) Determine the intersection of the geometric objects in (a) and (b) above. **[4]**

(d) Let A, B and C be the points with position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$.

Let P be the plane that passes through A, B and C.

(i) Find a vector perpendicular to P. **[6]**

(ii) Find the area of the triangle with vertices A, B and C. **[4]**

Question 4. Let $A = \begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) Either evaluate the following expressions or explain why they are meaningless:

(i) $2B$; **[2]**

(ii) $A - 2B$; **[2]**

(iii) A^2; **[4]**

(iv) B^2. **[2]**

(b) Find $\det(A)$. **[4]**

(c) Is A invertible? If A is invertible, then compute A^{-1}. Justify your answer, stating clearly the theorems you are using from the lecture notes. **[6]**

Question 5. Let $t : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$t\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ 3x+5y \end{pmatrix}.$$

(a) Prove that t is a linear transformation. **[6]**

(b) Find the corresponding matrix, A, for t. **[4]**

(c) Give a geometric description of $|\det(A)|$. **[4]**

(d) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f\begin{pmatrix} x \\ y \end{pmatrix} = xy.$$

Prove that f is not a linear transformation. **[6]**

End of Paper.