Student: ______________ **Date:** ______________	**Instructor:** Rainer Klages **Course:** MTH4101/MTH4201 Calculus II 2023	**Assignment:** Semester B final assessment 2023

1. Write an iterated triple integral in the order dx dy dz for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{6}+\frac{y}{8}+\frac{z}{7}=1$.

- A. $\int_0^7 \int_0^{1-y/8} \int_0^{1-y/8-z/7} dx\,dy\,dz$
- B. $\int_0^7 \int_0^{8(1-z/7)} \int_0^{6(1-y/8-z/7)} dx\,dy\,dz$
- C. $\int_0^7 \int_0^{1-z/7} \int_0^{1-y/8-z/7} dx\,dy\,dz$
- D. $\int_0^7 \int_0^{6(1-y/8)} \int_0^{6(1-y/8-z/7)} dx\,dy\,dz$

ID: 14.5-3

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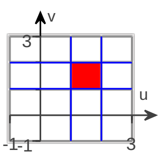
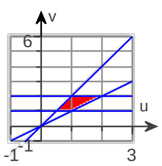
2. **a.** Find the Jacobian of the transformation $x=u$, $y=uv$ and sketch the region $G: 1 \le u \le 2, 1 \le uv \le 2$, in the uv-plane.

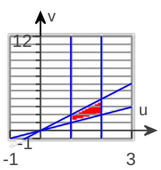
b. Then use $\iint_R f(x,y)\,dx\,dy = \iint_G f(g(u,v),h(u,v))|J(u,v)|\,du\,dv$ to transform the integral $\int_1^2 \int_1^2 \frac{y}{x}\,dy\,dx$ into an integral over G, and evaluate both integrals.

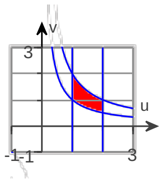
a. The Jacobian is ______.

Choose the correct sketch of the region G below.

A. 

B. 

C. 

D. 

b. Write the integral over G.

The integral is $\int_{\square}^{\square} \int_{\square}^{\square} \square \, dv\,du$.

Evaluate the integrals.

The evaluation for both integrals is ______. (Type an exact answer.)

ID: 14.8.10

3. Find the Taylor series generated by f at $x=a$.

$f(x)=2^x,\ a=1$

Choose the correct answer below.

- **A.** $2^x = \sum_{n=0}^{\infty} \frac{2(x-1)^n(\ln 2)^{n+1}}{n!}$
- **B.** $2^x = \sum_{n=0}^{\infty} \frac{2(x-1)^n(\ln 2)^{n}}{n!}$
- **C.** $2^x = \sum_{n=0}^{\infty} \frac{2(x-1)^{n+1}(\ln 2)^{n}}{n!}$
- **D.** $2^x = \sum_{n=0}^{\infty} \frac{2(x-1)^n}{(\ln 2)^n n!}$

ID: 9.8.32

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4. Find the equation for the tangent plane and the normal line at the point $P_0(1,2,3)$ on the surface $x^2+4y^2+3z^2=44$.

Using a coefficient of 1 for x, the equation for the tangent plane is ______.

Find the equations for the normal line. Let $x=1+2t$.

$x=$ ______, $y=$ ______, $z=$ ______

(Type expressions using t as the variable.)

ID: 13.6.1

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5. Find a formula for the nth term of the sequence where a_n is calculated directly from n.

$\frac{2}{1}, \frac{5}{2}, \frac{8}{6}, \frac{11}{24}, \frac{14}{120}, \ldots$

$a_n =$ ______ for $n \geq 1$

ID: 9.1.23

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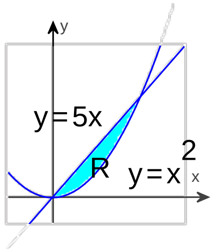
6. Write an iterated integral for $\iint_R dA$ over the region R described to the right using
a. vertical cross-sections,
b. horizontal cross-sections.



a. Write the correct iterated integral using vertical cross-sections. Select the correct answer below and fill in the answer boxes to complete your choice.

○ **A.** $\int_{\square}^{\square} \int_{\square}^{\square}$ dy dx

○ **B.** $\int_{\square}^{\square} \int_{\square}^{\square}$ dx dy

b. Write the correct iterated integral using horizontal cross-sections. Select the correct answer below and fill in the answer boxes to complete your choice.

○ **A.** $\int_{\square}^{\square} \int_{\square}^{\square}$ dx dy

○ **B.** $\int_{\square}^{\square} \int_{\square}^{\square}$ dy dx

ID: 14.2.11

7. Express the area of the region bounded by the given lines as an iterated double integral.

The lines $x = 0$, $y = 8x$, and $y = 3$

○ **A.** $\int_0^8 \int_0^{y/3} dx\,dy$

○ **B.** $\int_0^3 \int_0^{y/8} dx\,dy$

○ **C.** $\int_0^8 \int_0^{x/3} dy\,dx$

○ **D.** $\int_0^3 \int_0^{y/8} dy\,dx$

ID: 14.3-2

8. Change the Cartesian integral to an equivalent polar integral, and then evaluate.

$$\int_{-7}^{0} \int_{-\sqrt{49-x^2}}^{0} \frac{1}{1+\sqrt{x^2+y^2}}\, dy\, dx$$

- A. $\dfrac{\pi(7-\ln 8)}{4}$
- B. $\dfrac{\pi(7-\ln 8)}{2}$
- C. $\dfrac{\pi(7+\ln 8)}{4}$
- D. $\dfrac{\pi(7+\ln 8)}{2}$

ID: 14.4-3

9. Evaluate the double integral over the given region.

$$\iint_R \frac{1}{(x+1)(y+1)}\, dA, \quad R: 0 \le x \le 2,\ 0 \le y \le 5$$

- A. $6\ln 3$
- B. $\dfrac{1}{6}\ln 3$
- C. $\ln 3 \ln 6$
- D. $\ln 18$

ID: 14.1-14

10. Determine whether the series $\sum_{n=0}^{\infty} e^{-3n}$ converges or diverges. If it converges, find its sum.

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** The series converges because $\lim_{n\to\infty} e^{-3n} = 0$. The sum of the series is ____.
  (Type an exact answer.)
- **B.** The series diverges because it is a geometric series with $|r| \geq 1$.
- **C.** The series diverges because $\lim_{n\to\infty} e^{-3n} \neq 0$ or fails to exist.
- **D.** The series converges because $\lim_{k\to\infty} \sum_{n=0}^{k} e^{-3n}$ fails to exist.
- **E.** The series converges because it is a geometric series with $|r| < 1$. The sum of the series is ____.
  (Type an exact answer.)

ID: 9.2.59

11. Find the derivative of the function at the given point in the direction of **A**.

$f(x,y,z) = 4x - 8y + 2z$, $(-10,-3,-3)$, $\mathbf{A} = 3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$

- **A.** $\frac{32}{7}$
- **B.** $\frac{80}{7}$
- **C.** 8
- **D.** $\frac{24}{7}$

ID: 13.5-10

12. Evaluate $\frac{\partial w}{\partial u}$ at $(u,v) = (5,1)$ for the function $w(x,y) = xy - y^2$; $x = u - v$, $y = uv$.

- **A.** 4
- **B.** 9
- **C.** -1
- **D.** 6

ID: 13.4-4

13.

Define f(0,0) in a way that extends $f(x,y) = xy\frac{x^2 - y^2}{x^2 + y^2}$ to be continuous at the origin.

Let f(0,0) be defined to be [ ].

ID: 13.2.64

14. Assume that the recursively defined sequence converges and find its limit.

$a_1 = -20, \; a_{n+1} = \sqrt{20 + a_n}$

The sequence converges to [ ]. (Type an integer or a decimal.)

ID: 9.1.103

15. Find ∂f/∂x and ∂f/∂y.

$f(x,y) = 2x^{2y}$

$\frac{\partial f}{\partial x} =$ [ ]

$\frac{\partial f}{\partial y} =$ [ ]

ID: 13.3.19

16. For what values of x does the series converge conditionally?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n4^n}$$

- **A.** $x = 2$
- **B.** $x = -6$
- **C.** $x = -6, x = 2$
- **D.** None

ID: 9.7-25

17. Find the limit.

$$\lim_{\substack{(x,y)\to\left(\frac{81}{2},\frac{81}{2}\right)\\ x+y\neq 81}} \frac{x+y-81}{\sqrt{x+y}-9}$$

- A. 18
- B. 9
- C. 0
- D. There is no limit.

ID: 13.2-4

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18. Use any method to determine if the series converges or diverges. Give reasons for your answer.

$$\sum_{n=1}^{\infty} (9e)^{-n} n^4$$

Select the correct choice below and fill in the answer box to complete your choice.

(Type an exact answer.)

- A. The series diverges because the limit used in the nth-Term Test is ____.
- B. The series converges because the limit used in the nth-Term Test is ____.
- C. The series converges because the limit used in the Ratio Test is ____.
- D. The series diverges because the limit used in the Ratio Test is ____.

ID: 9.5.34

19. Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = \frac{9}{x^2 + y^2 - 1}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

○ **A.** A local maximum occurs at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)
The local maximum value(s) is/are ☐.
(Type an exact answer. Use a comma to separate answers as needed.)

○ **B.** There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

○ **A.** A local minimum occurs at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)
The local minimum value(s) is/are ☐.
(Type an exact answer. Use a comma to separate answers as needed.)

○ **B.** There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** A saddle point occurs at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)

○ **B.** There are no saddle points.

ID: 13.7.21

20. Given the function $f(x,y) = 4y - 6x$, answer the following questions.
a. Find the function's domain.
b. Find the function's range.
c. Describe the function's level curves.
d. Find the boundary of the function's domain.
e. Determine if the domain is an open region, a closed region, both, or neither.
f. Decide if the domain is bounded or unbounded.

a. Choose the correct domain of the function $f(x,y) = 4y - 6x$.

- **A.** All points in the first quadrant
- **B.** All points in the xy-plane except the origin
- **C.** $y \geq \frac{3}{2}x$
- **D.** All points in the xy-plane

b. Choose the correct range of the function $f(x,y) = 4y - 6x$.

- **A.** All non-negative integers
- **B.** All non-negative real numbers
- **C.** All integers
- **D.** All real numbers

c. Choose the correct description of the level curves of $f(x,y) = 4y - 6x$.

- **A.** Circles
- **B.** Ellipses
- **C.** Hyperbolas
- **D.** Straight Lines

d. Does the domain of the function $f(x,y) = 4y - 6x$ have a boundary?

- No
- Yes

e. Choose the correct description of the domain of $f(x,y) = 4y - 6x$.

- **A.** Neither open nor closed
- **B.** Closed Region
- **C.** Open Region
- **D.** Both open and closed

f. Is the domain of $f(x,y) = 4y - 6x$ bounded or unbounded?

- Bounded
- Unbounded

ID: 13.1.17

1.
B. $\int_{0}^{7} \int_{0}^{8(1-z/7)} \int_{0}^{6(1-y/8-z/7)} dx\,dy\,dz$

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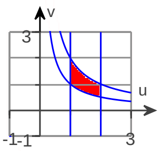
2. u

D. 

1

2

$1/u$

$2/u$

uv

$\frac{3\ln 2}{2}$

3.
B. $2^x = \sum_{n=0}^{\infty} \frac{2(x-1)^n (\ln 2)^n}{n!}$

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4. $x + 8y + 9z = 44$

$1 + 2t$

$2 + 16t$

$3 + 18t$

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5. $\frac{3n-1}{n!}$

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6. A. $\int_{0}^{5} \int_{x^2}^{5x} dy\,dx$

A. $\int_{0}^{25} \int_{\frac{y}{5}}^{\sqrt{y}} dx\,dy$

7.
B. $\int_{0}^{3} \int_{0}^{y/8} dx\,dy$

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8. B. $\frac{\pi(7-\ln 8)}{2}$

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9. C. $\ln 3 \ln 6$

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10. E.

The series converges because it is a geometric series with $|r|<1$. The sum of the series is $\boxed{\frac{e^3}{e^3-1}}$.

(Type an exact answer.)

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11. C. 8

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12. C. -1

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13. 0

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14. 5

---

15. $4yx^{2y-1}$

$4x^{2y}\ln x$

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16. A. $x=2$

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17. A. 18

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18. C. The series converges because the limit used in the Ratio Test is $\boxed{\frac{1}{9e}}$.

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19. A. A local maximum occurs at $\boxed{(0,0)}$.

(Type an ordered pair. Use a comma to separate answers as needed.)

The local maximum value(s) is/are $\boxed{-9}$.

(Type an exact answer. Use a comma to separate answers as needed.)

B. There are no local minima.

B. There are no saddle points.

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20. D. All points in the xy-plane

    D. All real numbers

    D. Straight Lines

    No

    D. Both open and closed

    Unbounded
