Queen Mary
University of London

Main Examination period 2017

MTH4101 / MTH4201: Calculus II

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Examiners: R. J. Harris, M. Luczak

Question 1.

(a) Calculate a_1, a_2, and a_3 for the sequence

$$a_n = \frac{n+(-1)^n}{2n}$$

and then obtain the limit of a_n as $n \to \infty$. [7]

(b) Find the sum of the series

$$\sum_{n=1}^{\infty} \left(\frac{2}{n^2} - \frac{2}{(n+1)^2} \right).$$ [7]

(c) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{e^x \sin y}{y}.$$ [7]

(d) For the function

$$f(x,y) = x\ln(xy) + y\cos x,$$

compute f_{xy} and f_{yx}, and show that these are equal. [7]

(e) Find all the local maxima, local minima, and saddle points of the function

$$g(x,y) = 2 + 4x - x^2 - 3y^2.$$ [7]

(f) Sketch the region of integration for

$$\int_0^9 \int_{y/3}^{\sqrt{y}} dx\, dy$$

and write an equivalent integral with the order of integration reversed. [7]

(g) Find the average value of $h(x,y,z) = 2x + 3y^2 - 8z^3$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 1$. [7]

(h) Solve the initial value problem

$$\frac{dy}{dx} = 4x^3 e^y, \quad y(0) = 0,$$

giving the solution in implicit form. [7]

Question 2.

(a) By calculating derivatives of the function

$$f(x) = \frac{1}{1+2x},$$

obtain its Maclaurin series, explicitly including terms up to x^3. State the range of x for which your series converges to $f(x)$. **[7]**

(b) Show how your result in (a) can be used to obtain the Maclaurin series for $\ln(1+2x)$ and give a formula for the nth term of this series. **[4]**

Question 3.

(a) Calculate the gradient vector of the function $g(x,y) = e^y \sin x$ at the point $(0,1)$. Use your result to calculate the directional derivative of this function at $(0,1)$ in the direction of the unit vector $\mathbf{v} = (1/\sqrt{2})\mathbf{i} + (1/\sqrt{2})\mathbf{j}$. **[6]**

(b) Now consider a general function $f(x,y)$. State the definition, in terms of a limit, for the derivative of f at $P_0(x_0,y_0)$ in the direction of the unit vector $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$. **[2]**

(c) Using the definition from part (b), evaluate the derivative of $g(x,y) = e^y \sin x$ at $(0,1)$ in the direction of the unit vector $\mathbf{w} = (1/\sqrt{2})\mathbf{i} - (1/\sqrt{2})\mathbf{j}$. **[3]**

Question 4.
Consider the function

$$f(x,y) = \frac{1}{(4-x^2-y^2)}.$$

(a) Integrate $f(x,y)$ over the disc $x^2+y^2 \leq 1$. **[8]**

(b) Does the integral of $f(x,y)$ over the disc $x^2+y^2 \leq 4$ exist? Give reasons for your answer. **[3]**

Question 5.

(a) Use the Integral Test to determine for which values of p the series

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^p}$$

converges, and for which it diverges. **[8]**

(b) Explain why the Ratio Test cannot be used to determine the convergence of this series. **[3]**

End of Paper.