## MTH6113 - MATHEMATICAL TOOLS FOR ASSET MANAGEMENT - 2021/22



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This quiz is not currently available

## INFORMATION

Problem 1 (consists of questions 1-2)
Do the following statements contradict the semi-strong form of the efficient market hypothesis?
Answer yes/no and then briefly explain your reasoning with no more than 50 words.

## QUESTION 1

Not yet answered Marked out of 5.00

Today's returns are positively correlated with tomorrow's returns.

## QUESTION 2

Not yet answered Marked out of 5.00

By taking a higher risk, we can achieve a higher expected return.

## QUESTION 3

## Problem 2 (consists of question 3 only)

One of the following figures shows the empirical returns of a stock since 1993. The other figure shows simulated returns using the lognormal model with parameters fitted to the empirical data.


Complete the following statements, such that they are true.
The daily return shown in Figure 1

Choose...

Choose...

## INFORMATION

Problem 3 (consists of questions 4-7)
For the next questions, the following investment opportunities are given based on their mean return $\mu$ and their standard deviation of the return $\sigma$ :

| Asset 1 | $\mu_{1}=10 \%$ | $\sigma_{1}=5 \%$ |
| :--- | :---: | :---: |
| Asset 2 | $\mu_{2}=15 \%$ | $\sigma_{2}=7 \%$ |
| Asset 3 | $\mu_{3}=15 \%$ | $\sigma_{3}=6 \%$ |
| Asset 4 | $\mu_{4}=20 \%$ | $\sigma_{4}=10 \%$ |
| Asset 5 | $\mu_{5}=20 \%$ | $\sigma_{5}=7 \%$ |

## QUESTION 4

[^0]$\left(\mu_{5}, \sigma_{5}\right)>\left(\mu_{2}, \sigma_{2}\right)$

Select one:
O True
O False

## QUESTION 5

Does the following pairwise dominance hold?
$\left(\mu_{3}, \sigma_{3}\right)>\left(\mu_{2}, \sigma_{2}\right)$

Select one:
O True
O False

## QUESTION 6

Not yet answered Marked out of 5.00

Let also the correlation between Asset 2 and Asset 4 be given as $\rho_{24}=-0.25$. Construct Asset 6 as the portfolio with equal parts in Assets 2 and 4 .
Compute the standard deviation of the returns of Asset 6.
(state the answer in decimals with four digits after the decimal point)

Answer:

QUESTION 7
Not yet answered Marked out of 10.00

We note that Asset 6 now dominates Asset 2, even though Asset 4 does not dominate Asset 2. Use this example to explain the benefits of diversification.

## information

## Problem 4 (consists of questions 8 to 10)

We consider the lognormal model with returns modelled as
$X \sim \mathcal{N}\left(\mu, \sigma^{2}\right)$.
Let $\mu=0.04$ and $\sigma=0.25$. We denote the density function of X as $f_{X}$ and the distribution function of X as $F_{X}$ (with the inverse $F_{X}^{-1}$ ).

## QUESTION 8

Which of the following values equals to the value at risk $\operatorname{VaR}_{99 \%}(\mathrm{X})$ ?
a. $-1 / f_{X}(0.01) \approx-0.6312$
O. $1 / f_{X}(0.99) \approx-856.3200$
( c. $-\mathrm{Fx}^{-1}(0.01) \approx 0.5416$
O d. $\quad F^{-1}(0.99) \approx 0.6216$

## QUESTION 9

Which of the following scaling properties holds for any random variable $X$ and $\alpha \in(0,1)$ ?

Select one:
a. $\quad \operatorname{VaR}_{\alpha}(2 X)=\operatorname{VaR}_{\alpha}(X)+1$
b. $\operatorname{VaR}_{0.5 \alpha}(X)=0.5 \operatorname{VaR}_{\alpha}(X)$
c. $\operatorname{VaR}_{\alpha}(X+1)=\operatorname{VaR}_{\alpha}(X)+1$
d. $\operatorname{VaR}_{\alpha}(2 X)=2 \operatorname{VaR}_{\alpha}(X)$
e. $\operatorname{VaR}_{\alpha+1}(X)=\operatorname{VaR}_{\alpha}(X)+1$
f. $\quad \operatorname{VaR}_{0.5 \alpha}(X)=\operatorname{VaR}_{\alpha}(X)-1$
g. $\operatorname{VaR}_{0.5 \alpha}(2 X)=\operatorname{VaR}_{\alpha}(X)$

## QUESTION 10

Explain your choice in one paragraph.

## QUESTION 11

Problem 5 (consists of questions 11 only)
Consider N assets in Sharpe's Single-Index model with $\mu_{0}=0, \alpha_{i}=0, \beta_{i}=1, \sigma_{i}=1, i=1, \ldots, N$, i.e. the asset returns are given as
$R_{i}=R_{M}+\varepsilon_{i}, i=1, \ldots, N$
with $R_{\mathrm{M}} \sim \mathcal{N}\left(\mu_{\mathrm{M}}, \sigma_{\mathrm{M}}^{2}\right)$ and $\varepsilon_{i} \sim \mathcal{N}(0,1)$ pairwise independent.
Consider a portfolio P with equal weights of each asset, i.e.
$R_{\mathrm{P}}=\sum_{i=1}^{N} R_{i} / N$.
We note that
$\mathbb{E}\left(R_{i}\right)=\mathbb{E}\left(R_{\mathrm{P}}\right)=\mu_{\mathrm{M}}$
and
$\operatorname{Var}\left(\boldsymbol{R}_{i}\right)=\sigma_{\mathrm{M}}^{2}+1$
$\operatorname{Var}\left(R_{\mathrm{P}}\right)=\sigma_{\mathrm{M}}^{2}+1 / N$
For a large number of assets N , use this result to explain the concepts of

- diversifiable risks,
- non-diversifiable risks
(respond in 2-4 sentences)


## information

## Problem 6 (consists of questions 12 to 15)

Consider a market where all assumptions of the CAPM (Capital Asset Pricing Model) hold with an interest rate $\mu_{0}=3 \%$. The expected return of the market portfolio is $\mu_{\mathrm{MP}}=8 \%$ and its standard deviation is $\sigma_{\mathrm{MP}}=6 \%$.
Consider an efficient portfolio P with $\beta=0.5$


## QUESTION 15

Not yet answered Marked out of 8.00

Consider a third portfolio, which optimises risk and return for your personal risk appetite, by maximising the function $\exp \left(\mu-7 \sigma^{2}\right)$. Compute the standard deviation of this optimal portfolio. (Note: return the result in its decimal form with three digits after the decimal point. E.g. $1.5 \backslash \%$ should be input as 0.015 )

## Answer:

## INFORMATION

## Problem 7 (consists of question 16 to 18)

## QUESTION 16

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Let two lotteries be given:
L_1={}{\begin{array}{cl}{100}&{\mathrm{ with probability 50%}}\\{-90}&{\mathrm{ with probability 50%.}}
and
L_2 ={{ll
Which lottery does a risk-averse investor strictly prefer and why? Fill the gaps in the following text:
Using the }\square\mathrm{ of the utility function u, we can show that
u(10)\gequ(5)>u(100)+u(-90)/2.
Therefore we can show that
\mathbb{E}(u(L_2)) > \mathbb{E}(u(L_1))
and see that the investor prefers
\begin{tabular}{|c|c|c|c|c|c|}
\hline convexity & convexity and monotonicity & concavity and monotonicity & positivity & monotonicity and positivity & concavity and positivity \\
\hline
\end{tabular}
```


## QUESTION 17

Which of the given functions 1-4 is a utility function for risk-seeking investors? Briefly explain your choice in the next question.

1. $u_{1}(x)=\exp (x)-1$
2. $u_{2}(x)=\log (x-1)$
3. $u_{3}(x)=x^{4}$
4. $u_{4}(x)=\sqrt{|x|}$

## QUESTION 18

Not yet answered Marked out of 5.00

Briefly explain your choice using no more than 100 words.


[^0]:    Does the following pairwise dominance hold?

