Queen Mary University of London

Main Examination period 2023 – January – Semester A

MTH6157: Survival Models

Duration: 2 hours

The exam is intended to be completed within **2 hours**. However, you will have a period of **4 hours** to complete the exam and submit your solutions.

For actuarial students only: This module also counts towards IFoA exemptions. For your submission to be eligible, **you must submit within the first 3 hours.**

You should attempt ALL questions. Marks available are shown next to the questions.

All work should be **handwritten** and should **include your student number**. Only one attempt is allowed – **once you have submitted your work, it is final**.

In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;

Examiners: C.Sutton, A.Baule

Question 1 [14 marks]. A pension scheme offers a pension on retirement at age 67 and a life assurance benefit on death before retirement. It is assumed that the force of mortality is constant in each 10-year age group as follows.

Ages x	20 to 29	30 to 39	40 to 49	50 to 59	60 to 69
μ_x	0.001	0.002	0.003	0.005	0.008

(a) What are the advantages and disadvantages of this simple mortality assumption? **[4]**

(b) Calculate the probability that a pension scheme member age 66 will collect their pension. **[3]**

(c) Calculate the probability that a pension scheme member age 28 will collect their pension. **[4]**

(d) Calculate the probability that the life assurance benefit will become payable before retirement for a member currently age 44 **[3]**

Question 2 [17 marks]. A university library is evaluating how long it should allow students to borrow books for. It collects data from a group of students who are studying a module on East Asian Political History. The number of books returned after different number of days is as follows.

Days	1	2	3	4	5
Number of books	25	28	49	39	22

In addition, one book was reported stolen after 3 days and 14 books were still not returned when the data collection exercise ended after five days.

(a) Calculate the Kaplan-Meier estimate of the survival function for the number of days before a book is returned to the library. **[9]**

(b) If the new library policy is to set the number of days students are allowed to borrow for to be such that 80% of all books in the trial were returned before that date, what should the library's new book borrowing terms be? **[2]**

(c) Why might the university be concerned about relying on this study alone for setting a new library policy? **[6]**

Question 3 [25 marks]. A life assurance company has been making losses in its annuity business which the actuary concludes is due to the use of an out-of-date survival model in premium calculations as investigations have shown a change in the proportion of policyholders who are smokers. The actuary seeks to analyse the mortality experience further by fitting a Cox's Proportional Hazard Model to the last year of policyholder data. The model fitted is

$$\lambda(t) = \lambda_0 \exp(\beta,\, z^T)$$

where λ_0 is the baseline hazard, z is a 1x3 covariate vector, and β is a 1x3 vector of regression coefficients, such that:

- z_1 is 1 for a smoker and 0 for a non-smoker
- z_2 is the age of the policyholder minus 60
- z_3 is 0 if the annuity is for a pension and 1 if it was bought for some other reason
- $\beta_1 = 0.423$, $\beta_2 = 0.62$, $\beta_3 = -0.3$.

(a) Explain precisely what types of selection are present in this annuity business. **[5]**

(b) Describe the policyholder that is represented by the baseline hazard. **[3]**

(c) Derive an expression for the survival function of a smoker age 74 who has the annuity as part of their pension. **[5]**

(d) If the probability that the person in (c) above survives two years is 0.98, calculate the probability that a 66 year old non-smoker who bought an annuity as part of their tax planning survives two years. **[7]**

(e) Another actuary suggests that the model would be just as effective without the z_3 covariate. Describe carefully how this hypothesis could be tested including what statistics you would calculate. **[5]**

Question 4 [18 marks]. The government statistical service in a country is looking to update published mortality tables for that country following changes in mortality experience during the COVID-19 pandemic. They complete an investigation using a Poisson model to produce raw estimates of the force of mortality at ages from 0 to 110. To begin with these are graduated by reference to the previous standard table which was produced in 2013.

(a) What are the advantages and disadvantages of this method of graduation in this specific circumstance? [**6**]

The statistical service is considering two functions of the previous standard table rates μ_x^s for this graduation:

(1) $\mu_x = \mu_x^s + a$
(2) $\mu_x = \mu_{x+b}^s$

(b) Explain how you would select which function to use and how you could then estimate the parameter a or b. [**6**]

(c) Explain how a graduation that uses the method of splines might overcome the disadvantages identified in (a). [**6**]

Question 5 [26 marks]. Upon completing the graduation by reference to the previous standard table referred to in Question 4 above, the government statistical service wish to test the quality of the graduation. The raw model estimates of the force of mortality, the graduated rates and the exposed to risk at ages 71 to 82 is as follows:

Age x	Exposed to risk	Model estimate	Graduated rate
71	2245	0.0142	0.0138
72	2134	0.0154	0.0145
73	2045	0.0156	0.0158
74	2004	0.0169	0.0165
75	1945	0.0195	0.0175
76	1904	0.0204	0.0196
77	1834	0.0215	0.0213
78	1783	0.0236	0.0227
79	1728	0.0268	0.0248
80	1649	0.0296	0.0286
81	1622	0.0328	0.0326
82	1594	0.0359	0.0357

(a) Complete a statistical test of the overall goodness-of-fit of this graduation at ages 71 to 82. [**8**]

It has been suggested that the graduation method chosen underestimates the impact of the COVID-19 pandemic at older ages.

(b) Name two statistical tests that could be used to evaluate this claim. [**2**]

(c) Complete these two statistical tests for ages 71 to 82. [**10**]

(d) What conclusions can you draw from the two tests in (c) about the claim that the graduation underestimates the impact of the pandemic at older ages? [**6**]

End of Paper.