Queen Mary
University of London

Main Examination period 2021 – January – Semester A

MTH5129: Probability & Statistics II

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

The exam is available for a period of **24 hours**. Upon accessing the exam, you will have **3 hours** in which to complete and submit this assessment.

When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about **2 hours** to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final**.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. For your submission to be eligible for IFoA exemptions, **you must submit within the first 3 hours of the assessment period**.

Examiners: N. Rodosthenous, A. Shestopaloff

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In this exam, $P(\cdot)$ denotes a probability measure defined on a space $(\Omega, \mathcal{F})$ and $E(\cdot)$ denotes the expectation with respect to P.

Question 1 [30 marks]. Suppose that X and Y have joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 12e^{-3x-4y} & \text{if } x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Verify that $f_{X,Y}$ is indeed a probability density function. [**5**]

(b) Find the marginal probability density function f_X and state the name of the distribution of X. [**8**]

(c) Find the conditional probability density function $f_{Y|X=x}$. [**8**]

(d) Are the random variables X and Y independent? Justify your answer. [**3**]

Suppose now that X and Y have joint probability density function given by

$$\widetilde{f}_{X,Y}(x,y) = \begin{cases} 12ye^{-3x-2y^2} & \text{if } x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(e) What is the probability $P(Y^2 > X > 0)$? [**6**]

Question 2 [9 marks]. Let X and Y have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{5}{32}x^4, & \text{for } 0 < x < 2,\ 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the joint probability density function $f_{U,V}$ of $U = X^2 - 1$ and $V = 2Y$. [**9**]

Question 3 [8 marks]. Suppose that X and Y are independent random variables each with a $\chi^2(2)$ distribution. Find the moment generating function of $U = X + 2Y$. State clearly and justify all steps taken. [**8**]
Hint: You may use without proof the fact that the moment generating function of a $\chi^2(\nu)$ *random variable* W *is*

$$M_W(t) = \left(\frac{1}{1-2t}\right)^{\nu/2}.$$

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Question 4 [25 marks]. In studying financial markets, it is often of interest to investigate distributions of various components of the order flow. Suppose we observe the following distribution of time intervals between order arrivals at an exchange (measured in milliseconds).

Time interval (milliseconds)	0-5	5-10	10-15	15-20	20-25	25+
Number of intervals	49	23	15	8	2	0

(a) Compute an estimate of the sample mean of time intervals between order arrivals. [**8**]

(b) Perform a goodness of fit test at the 5% significance level of the null hypothesis that the observed time intervals follow an Exponential distribution. [**13**]

(c) What is the p-value of your test in (b)? Does the p-value indicate that there is evidence against the null hypothesis? [**4**]

Question 5 [28 marks]. In a poll conducted during the 2016 Canadian federal election, 500 New Brunswick and 450 Saskatchewan voters were polled on their voting intention. Of the New Brunswick voters, 200 indicated a preference for the Liberal Party and 300 for the Conservative Party. Of the Saskatchewan voters, 175 indicated a preference for the Liberal Party and 275 for the Conservative Party.

(a) Test (at the 5% significance level) that the proportion of Conservative voters in Saskatchewan is greater than the proportion of Conservative voters in New Brunswick. Justify your choice of hypothesis testing procedure. [**8**]

(b) Test the hypothesis (at the 5% significance level) that the distribution of voting intentions in New Brunswick and in Saskatchewan is the same. [**8**]

(c) What is the p-value of your test in (b)? Does the p-value indicate that there is evidence against the null hypothesis? [**4**]

(d) Construct a 95% confidence interval for the difference in proportions of Liberal Party voters in New Brunswick and Liberal Party voters in Saskatchewan. [**8**]

End of Paper.