Queen Mary University of London

Main Examination period 2019

MTH6154 / MTH6154P: Financial Mathematics I

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: Dr. S. Muirhead, Prof. I. Goldsheid

Unless stated otherwise, give all numerical answers to **two decimal places**. Where the answer is a **rate**, this refers to the number of decimal places when written as a **percentage**, i.e. a rate of 0.02344 to two decimal places is 2.34% not 0.02.

Question 1. [28 marks] A bank offers interest at nominal rate r that is continuously compounded.

(a) Write down the effective rate r_{eff} in terms of r. **[3]**

(b) Suppose that $r = 2\%$. If you deposit £100 in an account today, how many years will it take for the account value to grow to £120? **[4]**

A creditor owes you three payments: (i) £150 due in 1 year's time, (ii) £250 due in 2 years' time, and (iii) £400 due in 4 years' time.

(c) Using the continuously compounded interest rate from part (b), calculate the present value and effective duration of the total credits owed. **[6]**

(d) Your creditor proposes paying you the total amount due, £800, in 3 years' time. By comparing present values, decide whether you should accept their offer. **[3]**

A second bank offers term deposits of 1 and 3 years' duration with respective spot rates s_1 and s_3.

(e) Explain briefly what is meant by the forward rate f_{t_1,t_2}. **[3]**

(f) Write down a formula for $f_{1,3}$ in terms of s_1 and s_3. **[4]**

A 3 year bond with face-value £10 million, redeemable at par, with 4% annual coupons is priced at £8 million.

(g) Show that the yield r of this bond is a solution to the equation:

$$20(1+r)^3 - (1+r)^2 - (1+r) - 26 = 0.$$ **[5]**

Question 2. [19 marks] An experiment has three possible outcomes A, B or C. You are offered odds on these outcomes of $o_A = 2$, $o_B = 3/2$ and $o_C = 3$.

(a) Suppose you place a bet of £2 on outcome B. Write down the return function of this bet. **[4]**

(b) Explain briefly what is meant by an arbitrage opportunity. **[3]**

(c) Determine whether the odds listed above give rise to an arbitrage opportunity. **[4]**

Recall that the Arbitrage Theorem states that, given a set of outcomes and a set of return functions over these outcomes, either (i) there exists an arbitrage opportunity or (ii) there exists a risk-neutral distribution over the outcomes.

(d) Explain briefly what is meant by a risk-neutral distribution. **[3]**

(e) You are offered two investments whose payoffs depend on which of two possible outcomes, X or Y, occurs:

> **Investment 1**, which pays £10 if outcome X occurs and £15 if outcome Y occurs.
>
> **Investment 2**, which pays £5 if outcome X occurs and £20 if outcome Y occurs.

If **Investment 1** costs £12, find the fair price of **Investment 2** assuming there is no arbitrage opportunity. **[5]**

Question 3. [13 marks] A share is currently priced at £80. You wish to model the future price of the share using a stochastic process.

(a) Give one reason why it would **not** be appropriate to model the share price as an i.i.d. sequence of random variables. **[3]**

You decide to model the year-on-year values $S(n)$ of the share price as a log-normal process with parameters $S = 80, \mu = 0.2$ and $\sigma^2 = 0.2$. You ask your broker to purchase a European call option on the share with strike $K = 140$ and expiry $T = 10$.

(b) Plot the payoff of this option as a function of the share price. **[4]**

(c) Find the probability that this option ends up 'out-of-the-money', i.e. has zero payoff. You may use the value of the standard normal c.d.f. $\Phi(1.0185) = 0.85$. **[6]**

Question 4. [25 marks] A share price is modelled via a two-period binomial model with parameters $S = 60$, $u = 3/2$, $d = 1/2$, and interest rate per time-period $r = 2\%$.

(a) Verify that the no-arbitrage assumption is valid in this model. **[4]**

(b) Show that the risk-neutral probabilities of up and down movements in the share price are
$$\tilde{p}_u = 0.52 \quad \text{and} \quad \tilde{p}_d = 0.48.$$ **[4]**

You are considering buying a European put option on the share with strike $K = 40$ and expiry $T = 2$.

(c) Find the no-arbitrage price at $t = 0$ of this option. **[9]**

Your stock broker advises you to consider buying an American put option.

(d) Explain the difference between an American put option and a European put option. **[3]**

(e) Find the no-arbitrage price at $t = 0$ of an American put option with strike $K = 200$ and expiry $T = 1$. **[5]**

Question 5. [15 marks] Recall that 'put-call parity' is an equation that relates the prices of put and call options written on the same share with identical strike prices and expiry times.

(a) Write down the formula for put-call parity in the case of continuously compounded interest. **[3]**

(b) Using a 'replicating portfolios' argument or otherwise, prove this formula. **[4]**

The Black–Scholes formula for a European call option with strike K and expiry T written on a stock with current price S, drift μ and volatility σ is

$$C = S\Phi(\eta) - Ke^{-rT}\Phi(\eta - \sigma\sqrt{T}),$$

where
$$\eta = \frac{\log \frac{S}{K} + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}.$$

(c) Prove that the Black–Scholes price of a call option is no more than the current stock price, i.e. $C \leq S$. **[4]**

(d) **Without** calculating a derivative, show that C is a non-increasing function of the strike K. **[4]**

End of Paper.