Main Examination period 2021 - May/June - Semester B
Online Alternative Assessments

## MTH5125: Actuarial Mathematics II

You should attempt ALL questions. Marks available are shown next to the questions.

## In completing this assessment:

- You may use books and notes.
- You may use calculators and computers, but you must show your working for any calculations you do.
- You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.
- You must not seek or obtain help from anyone else.

All work should be handwritten and should include your student number.
You have $\mathbf{2 4}$ hours to complete and submit this assessment. When you have finished:

- scan your work, convert it to a single PDF file, and submit this file using the tool below the link to the exam;
- e-mail a copy to maths@qmul.ac.uk with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about 2 hours to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. Only one attempt is allowed - once you have submitted your work, it is final.

IFoA exemptions. For actuarial students, this module counts towards IFoA actuarial exemptions. You are allowed two submissions for this exam - the first for your IFoA mark, and the second for your module mark. To be eligible for IFoA exemptions, your IFoA submission must be within the first 3 hours of the assessment period.

Examiners: C. Sutton, A. Baule

Question 1 [22 marks]. A life assurance company offers a whole life assurance to a life aged 50 with sum assured of $£ 68,000$ payable at the end of the policy year of death, secured by level annual premiums. It assumes AM92 Ultimate mortality and $4 \%$ per annum interest in all of its calculations. Gross premiums assume $£ 140$ initial and $£ 40$ final expenses. For policies sold through a distribution agreement with a supermarket chain the life assurer pays commission of $1.5 \%$ of each premium.
(a) Write down the equation of value for a policy sold to a supermarket customer and evaluate the office premium.
(b) For policies sold direct to customers via its website, the life assurer pays no commission but sends new policyholders a $£ 200$ voucher for a well-known online bookstore and allows for an additional $2 \%$ of each premium in expenses to cover direct marketing costs and website technology upgrades. Should a new policyholder buy direct online or via the supermarket?
(c) What is the minimum premium the life assurer should charge to online customers for the probability of a loss on the policy to be less than 0.025 ?

Question 2 [ 15 marks]. Life X is assumed to have a constant force of mortality of 0.0035 and life Y is assumed to have a constant force of mortality of 0.0025 . The interest rate is $1.4 \%$ per annum and benefits are assumed to be payable immediately on death.
(a) Calculate the single premium required for a Whole Life Last Survivor Assurance where the sum assured is $£ 15,000$ with $10 \%$ sales commission and termination expenses of $£ 750$.
(b) What additional information would you need to calculate the reserve for this policy 5 years later?

Question 3 [20 marks]. Alexis and Benji, both age 55 have been partners in a successful business making flutes and clarinets for professional musicians in leading orchestras around the world. Benji is worried about what will happen to his income if Alexis dies and so is willing to pay $12 \%$ of this year's earnings for some contingent life assurance. A life assurance company offers Benji two options:

Option 1: A reversionary annuity of $£ 40,000$ per annum payable annually in arrears with Alexis as the counter life.
Option 2: A contingent whole life assurance with sum assured of $£ 250,000$ payable at the end of the year of death if Alexis dies before Benji.
(a) If Benji evaluates these options assuming $4 \%$ per annum interest and mortality such that $\ddot{a}_{55}=14.543$ and $\ddot{a}_{55: 55}=12.947$, which option should he choose?
(b) What assumptions underlie your calculation in (a) and how realistic are these?
(c) If the life assurance company assumes that the present value of future expenses is $2 \%$ of the present value of future benefits, write down a formula for the gross premium reserve 8 years after inception of the policy you selected in (a) above assuming both Alexis and Benji are still alive.
(d) Explain what will happen to the gross premium reserve immediately after the first death.

Question 4 [23 marks]. An insurance company offers a policy that pays benefits on death or sickness within a three-year term. The death benefit is a sum assured of $£ 1,500$ payable immediately and the sickness benefit is $£ 300$ payable immediately on diagnosis of an illness listed on the policy schedule plus a further $£ 100$ paid 18 months later. Forces of decrement are given in the table below.

| Age | force of <br> mortality | force of <br> sickness |
| :---: | :---: | :---: |
| 40 | 0.0025 | 0.0044 |
| 41 | 0.0027 | 0.0045 |
| 42 | 0.0031 | 0.0047 |

(a) Construct a multiple decrement table at ages $40,41,42$ and 43 with a radix of 10,000 using the forces of decrement above.
(b) Use the table you construct in (a) to calculate a single premium for the insurance policy assuming $2 \%$ per annum interest and expenses of $8 \%$ of the single premium.
(c) Why might the approach in (a) and (b) above underestimate the cost of the death benefit?
(d) If the insurance company wishes to add a survival benefit equal to the return of $50 \%$ of the single premium at the end of the policy term if no death or sickness benefit has been paid, what is the percentage increase in single premium required assuming the same basis as (b) above?

Question 5 [20 marks]. Annual premiums are paid into a 3 year unit linked endowment policy where $98 \%$ of each premium is allocated to units in a fund that carries a $3 \%$ bid-offer spread and charges management fees of $0.75 \%$ of assets at the end of each policy year. The policy has a death benefit of the bid value of units payable at the end of the year of death subject to a minimum of $£ 12,500$. The survival benefit is the bid value of units at the end of the term. The life assurance company estimates that expenses are $£ 95$ per policy per year.
(a) Produce projected revenue accounts for each year of a policy with an annual premium of $£ 5,000$ assuming the annual rate of mortality is 0.00498 , an investment return of $6.9 \%$ per annum and an interest rate on cash balances of $3 \%$ per annum.
(b) Does the life assurance company meet its internal profit margin objective of $5 \%$ on this policy if its risk discount rate is $5.5 \%$ per annum?

End of Paper.

