Queen Mary University of London

B. Sc. Examination by course unit 2015

MTH 6139: Time Series

Duration: 2 hours

Date and time: 01.06.2015, 10.00-12.00

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Examiner(s): B. Bogacka

Question 1. (20 marks)

Let $\{X_t\}_{t=1,2,\ldots}$ be a time series such that

$$X_t = \beta_0 + \beta_1 t + \beta_2 t^2 + Y_t,$$

where $\beta_0, \beta_1, \beta_2$ denote unknown constant parameters and Y_t denotes a zero-mean weakly stationary process.

(a) Describe briefly two methods of removing trend from such a time series: the differencing method and the method of least squares estimation of the unknown parameters. What is the main advantage and disadvantage of each of these two methods? [**6**]

(b) Show that $\nabla^2 X_t$ is a weakly stationary process and give its autocovariance function in terms of the process Y_t. [**7**]

(c) Give the definition of the convolution operation on the linear filters $\{a_j\}$ and $\{b_k\}$. [**2**]

(d) Show that the operator ∇^2 is a convolution of two filters of the form $(-1, 1)$. [**5**]

Question 2. (20 marks)
A time series data set of size $n = 100$ was modelled as an $AR(2)$ process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + Z_t, \quad Z_t \sim WN(0, \sigma^2).$$

(a) Give the Yule-Walker equations for estimation of the AR parameters and of the variance of the white noise. Briefly explain your notation. [**5**]

(b) Find the estimates of the model parameters ϕ_1, ϕ_2 and σ^2 knowing that the estimates of the series' variance and the autocorrelation at lags 1 and 2 are

$$\widehat{\gamma}(0) = 1.6, \quad \widehat{\rho}(1) = 0.6, \quad \widehat{\rho}(2) = 0.4.$$

[**6**]

(c) Calculate 95% confidence intervals for ϕ_1 and for ϕ_2.
Note that for a standard normal random variable U and for u_α such that $P(|U| > u_\alpha) = \alpha$ we have $u_\alpha = 1.96$ for $\alpha = 0.05$. [**7**]

(d) Predict value x_{101} of the series knowing that $x_{100} = 2.4$ and $x_{99} = 1.6$. [**2**]

Question 3. (20 marks)
Consider the ARMA$(1,1)$ model of a Time Series $\{X_t\}_{t=1,2,\dots}$,

$$X_t - \phi X_{t-1} = Z_t + \theta Z_{t-1},$$

where ϕ and θ denote unknown constant parameters, $\phi + \theta \neq 0$ and Z_t is a white noise random variable.

(a) What conditions do parameters ϕ and θ have to meet for X_t to be causal and invertible? Explain what it means for a time series to be causal and invertible. [**4**]

(b) Give the definition of a linear process. [**4**]

(c) Use the linear process form of an ARMA(1,1) time series, which is

$$X_t = Z_t + (\phi + \theta) \sum_{j=1}^{\infty} \phi^{j-1} Z_{t-j}$$

to obtain the autocorrelation function of ARMA(1,1) at lag 1. [**12**]

Question 4. (20 marks)
Write down a general model for each of the following series. Use the backshift operator and explain your notation.

(a) ARMA$(1,1)_{12}$ [**4**]

(b) MA$(2)_4$ [**4**]

(c) AR(1) [**4**]

(d) ARIMA$(2,0,1) \times (0,1,0)_{12}$ [**4**]

(e) ARIMA$(2,1,0)$ [**4**]

Question 5. (20 marks)
Consider the seasonal model AR$(1)_4$ given by

$$X_t = \Phi X_{t-4} + Z_t,$$

where $Z_t \sim WN(0, \sigma^2)$ and $|\Phi| < 1$.

(a) Using the method of matching coefficients obtain the linear form of the series X_t. [**12**]

(b) Hence, show that the autocorrelation function of the series is [**8**]

$$\rho(\tau) = \begin{cases} 1 & \text{if } \tau = 0, \\ \Phi^k & \text{if } \tau = 4k, k = 1, 2, \dots, \\ 0 & \text{otherwise.} \end{cases}$$

End of Paper.