Queen Mary
University of London

B. Sc. Examination by course unit 2015

MTH6121: Introduction to Mathematical Finance

Duration: 2 hours

Date and time: 29th April 2015, 10:00–12:00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and **cross through any work that is not to be assessed**.

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Exam papers must not be removed from the examination room.

Examiner(s): N. Rodosthenous

Question 1. Let the probability that the FTSE100 index increases today be 0.6 and the probability it goes up tomorrow be 0.5. Also, let the probability that it increases both today and tomorrow be 0.2. What is the probability

(a) it increases today but does not increase tomorrow? [**3**]

(b) it does not increase tomorrow, given that it increases today? [**3**]

(c) it increases today or tomorrow, but does not go up on both days? [**3**]

(d) it increases neither today nor tomorrow? [**3**]

Question 2. Let X be a continuous random variable with probability density function f_X given by

$$f_X(x) = \begin{cases} e^{-x} & \text{if } x \geq 0; \\ 0 & \text{if } x < 0. \end{cases}$$

(a) Verify that f_X is a proper probability density function. [**3**]

(b) Find the first moment of X. [**3**]

(c) Using the Transformation of random variables theorem, find the probability density function of the random variable Y, defined by $Y = 5X^3$. [**6**]

Question 3. An analyst working in a financial institution found that the monthly share price of company A can be described by an IID lognormal model with $\mu = 0.02$ and $\sigma^2 = (0.11)^2$.

(a) Explain what is meant by a Lognormally distributed random variable with parameters μ and σ^2, and what values can μ and σ take? [**3**]

(b) What does the IID lognormal model assume for the relative share price changes? [**2**]

(c) Determine the probability that the share price of company A is lower at the end of the 10th month than at the end of the 2nd month. [**6**]

Question 4.

(a) What is a *stochastic process*? [**3**]

(b) What is a *Wiener process* or *Brownian motion*? [**3**]

(c) Let $Y(t)$ be Brownian motion with drift having drift parameter $\mu = 0.3$ and volatility parameter $\sigma = 0.5$. What distribution does $Y(t)$ follow? [**2**]

(d) Determine $E(Y(5) + Y(7))$. [**2**]

(e) Determine $\mathrm{Cov}(Y(5), Y(7))$. **[3]**

(f) Determine $\mathrm{Var}(Y(5) + Y(7))$. **[3]**

Question 5. For this problem assume that the nominal annual interest rate (compounded monthly) is 1%.

(a) What is the present value β of 1 pound received in 1 month from today (discounting factor)? Give your answer in 8 decimal places. **[2]**

(b) Suppose that you buy a car for $10,000$ pounds. The seller gives you a deal where you initially pay $2,500$ pounds and then make a payment A at *the end* of each month for 5 years until your debt is paid off. What is the payment A? **[5]**

Question 6.

(a) What is an arbitrage opportunity? **[3]**

(b) State the Law of one price. **[4]**

(c) Suppose that you own a UK-based company having a liability of $\$\,1{,}000{,}000$ in half a year. Consider the following two different strategies to achieve this:

(A) In order to hedge the foreign exchange risk of your company (since you do not know the future exchange rate), you decide to go long a forward contract to buy $\$\,1{,}000{,}000$ for a (fixed) forward price $\pounds F$ in half a year.

(B) Purchase (in £) the amount $\$\,e^{-r_{\$}T}\,1{,}000{,}000$ at time 0 (when you know the exchange rate).

Suppose that the continuously compounded nominal interest rate in the UK is $r_{\pounds} = 1\%$ and in the US is $r_{\$} = 2\%$, while the exchange rate at time 0 is $0.67\pounds/\$$. What is the fair value of F? **[7]**

Question 7.

(a) Explain what is a European call option. **[5]**

(b) Assume that the interest rate is r per time period. Suppose that at time 0, BP share price is traded at $\pounds 425$, and that at the next time period 1, it is either traded at $\pounds 415$ or $\pounds 438$. We consider a European call option on the BP share with maturity time period 1 and strike price $\pounds 428$. What is the no-arbitrage price C of this call option?
Hint: *Use the Arbitrage Theorem with two wagers; purchase one share and purchase one call option.* **[6]**

Question 8. Suppose that the share price of company B is currently trading at $\pounds 20$ on the London Stock Exchange. The annual share price evolves according to a geometric Brownian motion with drift parameter $\mu = 0.7$ and volatility parameter $\sigma = 1.2$. Suppose also that the continuously compounded interest rate is 4%.

(a) What is the share price at time t, $S(t)$, equal to, under the above assumptions? [**2**]

(b) Find the probability that after 5 weeks the share has at least doubled its value. [**5**]

(c) What would change in the evolution of the share price, if it followed the risk-neutral geometric Brownian motion? [**2**]

(d) Suppose that there exists a European call option written on the share price of company B with strike price K and and maturity time T. Write down the Black-Scholes formula for the price of this option. [**3**]

(e) What is the probability that you will exercise this call option with strike price $K = \pounds 22$ at maturity $T = 4$ months? [**5**]

End of Paper—An appendix of 1 page follows.

Table of the cumulative standard normal distribution

$$\Phi(x) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-t^2/2}\,\mathrm{d}t.$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

End of Appendix.