Main Examination period 2021 – May/June – Semester B
Online Alternative Assessments

MTH6018: Coding Theory

You should attempt ALL questions. Marks available are shown next to the questions.

In completing this assessment:

- **You may use books and notes.**
- **You may use calculators and computers, but you must show your working for any calculations you do.**
- **You may use the Internet as a resource, but not to ask for the solution to an exam question or to copy any solution you find.**
- **You must not seek or obtain help from anyone else.**

All work should be **handwritten** and should **include your student number**.

You have **24 hours** to complete and submit this assessment. When you have finished:

- scan your work, convert it to a **single PDF file**, and submit this file using the tool below the link to the exam;
- e-mail a copy to **maths@qmul.ac.uk** with your student number and the module code in the subject line;
- with your e-mail, include a photograph of the first page of your work together with either yourself or your student ID card.

You are expected to spend about 2 hours to complete the assessment, plus the time taken to scan and upload your work. Please try to upload your work well before the end of the submission window, in case you experience computer problems. **Only one attempt is allowed – once you have submitted your work, it is final.**

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Question 1 [15 marks].
Consider the two codes $\mathcal{C}$ and $\mathcal{D}$ over the alphabet $\mathbb{A} = \{\text{A, B, } \ldots, \text{ Y, Z}\}$ given by:

$$\mathcal{C} = \{\text{FRANCE, GREECE, LATVIA, SERBIA, SWEDEN}\}$$

and

$$\mathcal{D} = \{\text{ANGOLA, GAMBIA, MALAWI, UGANDA, ZAMBIA}\}.$$

(a) Compute the minimum distance of $\mathcal{C}$ and the minimum distance of $\mathcal{D}$. Include appropriate justifications for your answers. **[9]**

(b) For which values of $t > 0$, if any, are these two codes t-error-correcting? For which values are they t-error-detecting? **[4]**

(c) Are the two codes $\mathcal{C}$ and $\mathcal{D}$ equivalent? Justify your answer. **[2]**

Question 2 [15 marks]. Decide which of the following inequalities are true and which are false. Justify your answer in each case, stating which results from the course you use in your answer (if any).

(a) $A_3(5, 5) \geq 4$ **[5]**

(b) $A_2(23, 18) \geq 3$ **[5]**

(c) $A_2(10, 5) \leq 50$ **[5]**

Question 3 [17 marks].
Recall that the **rate** of a q-ary (n, M, d)-code $\mathcal{C}$ is defined to be the quantity

$$R(\mathcal{C}) = \frac{\log M}{n \log q}.$$

(a) Let $\mathbb{A}$ be a q-ary alphabet and $\mathcal{D} \subseteq \mathbb{A}^n$ a q-ary (n, M, d)-code, where $d \geq 2$. Is it possible that we could have $R(\mathcal{D}) = 1$? Justify your answer. **[3]**

(b) If $\mathcal{E}$ is a linear $[n, k]$-code over $\mathbb{F}_q$, what is the rate of $\mathcal{E}$? More generally, which numbers in the range 0 to 1 can be the rate of a **linear** code of length n over $\mathbb{F}_q$? Justify your answer. **[5]**

(c) Let $\mathbb{A}$ be a q-ary alphabet and $\mathcal{C}_1 \subseteq \mathbb{A}^n$ a q-ary (n, M, d)-code. Given two words $u = u_1 u_2 \cdots u_n$ and $v = v_1 v_2 \cdots v_n$ in $\mathcal{C}_1$, let $u||v$ denote the word $u_1 u_2 \cdots u_n v_1 v_2 \cdots v_n$. Define a new code by $\mathcal{C}_2 = \{u||u \colon u \in \mathcal{C}_1\}$.

(i) Show that $R(\mathcal{C}_2) = \frac{1}{2} R(\mathcal{C}_1)$. **[5]**

(ii) Show that $d(\mathcal{C}_2) = 2d(\mathcal{C}_1)$. **[4]**

Question 4 [12 marks].
Consider the code $\mathcal{C} = \{000, 010, 101, 110\}$ over the alphabet $\mathbb{A} = \{0, 1\}$. Construct a nearest-neighbour decoding process for $\mathcal{C}$ and compute the word error probability for the word 110 assuming that the symbol error probability p is equal to $\frac{1}{7}$. Show all your working. **[12]**

Question 5 [11 marks].

(a) Put the generator matrix over $\mathbb{F}_3$ given by

$$G = \begin{pmatrix} 2 & 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 2 & 1 \\ 1 & 1 & 2 & 0 & 2 \end{pmatrix}$$

into standard form, making it clear at each stage which operations are being used. **[6]**

(b) Give an example of a linear $[6, 3, 2]$-code over $\mathbb{F}_2$ for which there does not exist a generator matrix in standard form, and explain why it does not have such a generator matrix. **[5]**

Question 6 [15 marks].

(a) Let $\mathbb{F}_q$ be a finite field and let $\mathcal{C}$ be the repetition code of length n over $\mathbb{F}_q$. Show that $\mathcal{C}$ is a linear code. What are the dimension and redundancy of $\mathcal{C}$? **[4]**

(b) Consider the linear code $\mathcal{C} = \{000, 211, 122, 001, 002, 212, 210, 120, 121\}$ over $\mathbb{F}_3$.

(i) Write down a generator matrix for $\mathcal{C}$. **[3]**

(ii) Construct a Slepian array for $\mathcal{C}$ and use it to decode the word 111. **[8]**

Question 7 [15 marks].

(a) (i) Write down a parity-check matrix for the code $\mathrm{Ham}(2, 7)$. **[3]**

(ii) Using your parity-check matrix, explain why the minimum distance of $\mathrm{Ham}(2, 7)$ is exactly 3. **[4]**

(b) Consider the linear code $\mathcal{C}$ over $\mathbb{F}_5$ with the following parity-check matrix:

$$\begin{pmatrix} 3 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{pmatrix}.$$

(i) Decide whether or not $\mathcal{C}$ is an MDS code. Justify your answer. **[5]**

(ii) Suppose that the above matrix is instead taken to be the generator matrix of a linear code $\mathcal{D}$ over $\mathbb{F}_7$. Is $\mathcal{D}$ an MDS code? Justify your answer. **[3]**

End of Paper.