Queen Mary
University of London

Main Examination period 2018

MTH6108 / MTH6108P: Coding Theory

Duration: 2 hours

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You should attempt ALL questions. Marks available are shown next to the questions.

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Complete all rough work in the answer book and cross through any work that is not to be assessed.

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Examiners: W. Mannan, I. Tomašić

Question 1. [25 marks]

(a) Give the definitions of the following:

(i) a **code** of length n over an alphabet $\mathbb{A}$; **[2]**

(ii) the **distance** between two words of length n over $\mathbb{A}$; **[2]**

(iii) the **minimum distance** of a code; **[2]**

(iv) a q**-ary** (n,M,d)**-code**; **[2]**

(v) $A_q(n,d)$. **[2]**

(b) What does it mean to say that a code is t**-error-detecting**? **[2]**

(c) What does it mean to say that a code is t**-error-correcting**? **[3]**

(d) Suppose the minimum distance of a code is 9. What is the largest value of t for which the code is:

(i) t-error-detecting? **[2]**

(ii) t-error-correcting? **[3]**

(e) Suppose C is a $(9,7,4)$–code over the alphabet $\mathbb{F}_2$. An array is constructed with a row $R(\{u,v\})$ of length 9 for each pair of distinct words $u,v \in C$. The i-th entry of $R(\{u,v\})$ is given by $R(\{u,v\})_i = u_i + v_i$. Let x be the number of ones in this array. Show that:

$$84 \leq x \leq 108.$$ **[5]**

Question 2. [25 marks]

(a) (i) What is a **linear code** of length n over $\mathbb{F}_q$? **[2]**

(ii) What is a linear $[n,k,d]$-code over $\mathbb{F}_q$? **[2]**

(iii) What is a **generator matrix** of a linear code? How many rows and columns does a generator matrix of an $[n,k,d]$-code have? **[3]**

(b) Let C be a linear $[n,k,d]$-code over $\mathbb{F}_q$.

(i) What is the size of C? Explain your reasoning. **[4]**

(ii) State the **Singleton bound** for general (not necessarily linear) codes. **[3]**

(iii) State the **Singleton bound for linear codes**, relating the numbers n, k, d. Prove your statement, using parts (i) and (ii) of this question. **[4]**

(c) Let C be the linear code of length 3 over $\mathbb{F}_3$ spanned by the words 120, 210, 101, 011. Find a generator matrix of C. **[3]**

(d) Let D be the linear code given by

$$D = \{v \in \mathbb{F}_2^4 : v_1 + v_2 + v_3 + v_4 = 0\}.$$

Find a generator matrix for D. **[4]**

Question 3. [25 marks]

(a) Suppose C is a linear $[n,k]$-code over $\mathbb{F}_q$.

  (i) What is the **dual code** $C^{\perp}$? **[2]**

  (ii) What is a **parity-check matrix** for C? **[2]**

  (iii) Suppose H is a parity-check matrix for C. State the **Minimum Distance Theorem for Linear Codes**, which explains how the minimum distance of C is related to the linear independence of the columns of H. **[3]**

  (iv) What is the **syndrome** of a word $v \in \mathbb{F}_q^n$? **[3]**

  (v) Explain how to construct a **syndrome look-up table** for C. **[3]**

  (vi) Explain how to construct a nearest-neighbour decoding process for C using a syndrome look-up table. **[3]**

(b) Consider the binary code C with generator matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

  (i) Write down a parity-check matrix for C. **[2]**

  (ii) Construct a syndrome look-up table for C and use it to decode the word 1110. **[7]**

Question 4. [25 marks]

(a) Give the definition of a **perfect** code? **[3]**

(b) State conditions on n,k,d for an $[n,k,d]$-code to be a maximum distance separable (MDS) code. **[3]**

(c) Define the **q-ary Hamming code** $\mathrm{Ham}(r,q)$ for $r > 0$ and a prime power q. **[4]**

(d) Let $C = \mathrm{Ham}(2,5)$ be the Hamming code with parity-check matrix

$$\begin{bmatrix} 1 & 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

  (i) Find the minimum distance $d(C)$. Explain your reasoning. **[4]**

  (ii) Prove that C is perfect. **[4]**

  (iii) Determine $A_5(6,3)$. Explain your reasoning. **[4]**

  (iv) Is C an MDS code? Explain your reasoning. **[3]**

End of Paper.