Queen Mary
University of London

MTH6100 / MTH6100P: Actuarial Mathematics

Duration: 2 hours

Date and time: 09 May 2016, 14:30h–16:30h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt ALL questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. Please state on your answer book the name and type of machine used.

This is an OPEN BOOK exam

permitted:	**any printed material, e.g. books**
	any handwritten notes
	photocopies of any kind
prohibited:	**using communication devices, e.g. laptops or mobile phones**
	sharing material with other students

Exam papers must not be removed from the examination room.

Examiner(s): W. Just, D. Stark

Question 1. Give all monetary answers in this question to the nearest penny.

(a) Professor Onestone invests £500 in a savings account with a constant effective interest rate of 3% per annum. Find the accumulation after six years. [**3**]

(b) (i) State the relationship between the nominal rate of interest compounded p-thly, $i^{(p)}$, and the nominal rate of discount compounded p-thly, $d^{(p)}$. [**1**]

(ii) If the nominal interest rate is 8% per annum compounded quarterly, how much should be paid **in advance** to borrow £2,000 for three months? [**3**]

(c) Ms Chang has a debt of £4,000 due in five years' time and a debt of £7,000 due in eight years' time. There is a constant effective interest rate of 8% per annum.

(i) If Ms Chang is allowed to discharge these debts by making a single payment of £10,000, find the time at which that payment should be made. Give your answer to the nearest month. [**4**]

(ii) If instead she makes equal quarterly repayments in advance, find the premium necessary to pay off the debts in nine years. [**5**]

(d) Assume that the basic unit of time is 1 year and that the basic unit of money is £1. Write down the correct actuarial symbol for the present value of a series of monthly payments of £50 each, paid in perpetuity with the first payment in exactly one month's time. [**2**]

Question 2.

(a) (i) Explain in words the meaning of the life-table function ${}_tp_x$ and express it in terms of the survival function $s(x)$. [**2**]

(ii) Assuming the mortality given by table A1967-1970 select values, calculate the probability ${}_5p_{[42]}$ to 4 decimal places. [**4**]

(b) Suppose that a newborn has lifetime distributed according to a random variable X with survival function $s(x) = P(X > x)$.

(i) Show that the curtate further lifetime $K(x)$ satisfies

$$P(K(x) = k) = \frac{s(x+k) - s(x+k+1)}{s(x)}.$$

[**5**]

(ii) Show that the expected curtate further lifetime e_x satisfies

$$e_x = \frac{1}{l_x} \sum_{k=1}^{\infty} l_{x+k}$$

[**6**]

(c) Suppose that a population is subject to mortality described by ELT12. A group of 12000 newborns were born in a specific year. Estimate, to 4 significant figures, the number of survivors to age 40 from the group. [**3**]

Question 3. Let X be the exact age-at-death of a newborn and let $s(x) = P(X > x)$ be the corresponding survival function.

(a) Express $s(x)$ in terms of $\mu(x)$, the force of mortality at age x. [**2**]

(b) Zephirs are a species of apocryphal creature with $\mu(x)$ given by

$$\mu(x) = \frac{1}{2(9-x)} \qquad \text{for } 0 \leq x < 9.$$

(i) Show that

$$s(x) = \frac{\sqrt{9-x}}{3} \qquad \text{for } 0 \leq x < 9.$$

What does this imply about $s(x)$ for $x \geq 9$? [**7**]

(ii) Use this expression for the survival function to determine the probability that a zephir of age 5 dies by age 8. Give your answer as a fraction. [**4**]

(iii) Calculate the curtate expectation of life for a zephir of age 4. Give your answer to two decimal places. [**7**]

Question 4. On his 55th birthday, Dr Flitwick takes out a whole-life assurance policy with death benefit of £50,000 to be paid at the end of the year of death. Assume an effective annual interest rate of 4% and the mortality given by table A1967-70 **select** values. Give your answers to the nearest penny.

(a) Find the cost of this policy. [**7**]

(b) Calculate the annual premium required to purchase the policy if the premium is to be paid annually in advance for life. [**5**]

(c) Suppose that Dr Flitwick surrenders the policy on his 60th birthday, just before making the payment due on that date. Find the surrender value. [**8**]

Question 5.

(a) Mae West invests £1000 in a financial product which gives her £500 at the end of the first year and £200 at the end of each subsequent year. If the AER is 12%, determine the discounted payback period as well as the payback period of her investment. [**6**]

(b) In return for an investment today of £1,000, an investor is to receive £400 one year from now and £800 two years from now. Determine the yield on this transaction. Give your answer as a percentage to 2 decimal places. [**5**]

(c) Consider an investor who wishes not to accumulate money but to receive income while keeping her capital fixed at C. At time 0, she deposits £C at Big British Bank, which pays interest according to force of interest $\delta(s)$. That is, the income she earns is a continuous cashflow with rate $\rho(t) = C\delta(t)$, where $\delta(t)$ is the force of interest, and the amount she has in the bank is fixed at C. At time $T > 0$, she takes out her capital from the bank. Show that, at that time, the present value of her income and her capital equals C. [**6**]

(d) Show that

$$(\bar{I}\bar{a})_{\overline{n}|} = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}.$$

[**5**]

End of Paper—An appendix of 7 pages follows.

Age x	l_x	d_x	P_x	q_x	μ_x	$\overset{0}{e}_x$	Age x
0	100000	2449	.97551	.02449		68.09	0
1	97551	153	.99843	.00157	.00210	68.80	1
2	97398	96	.99901	.00099	.00134	67.90	2
3	97302	67	.99931	.00069	.00079	66.97	3
4	97235	60	.99938	.00062	.00063	66.02	4
5	97175	55	.99943	.00057	.00059	65.06	5
6	97120	51	.99948	.00052	.00054	64.09	6
7	97069	47	.99952	.00048	.00050	63.13	7
8	97022	43	.99956	.00044	.00046	62.16	8
9	96979	40	.99959	.00041	.00043	61.18	9
10	96939	38	.99961	.00039	.00040	60.21	10
11	96901	37	.99962	.00038	.00039	59.23	11
12	96864	37	.99962	.00038	.00038	58.25	12
13	96827	40	.99959	.00041	.00039	57.28	13
14	96787	45	.99953	.00047	.00043	56.30	14
15	96742	57	.99941	.00059	.00052	55.33	15
16	96685	75	.99922	.00078	.00067	54.36	16
17	96610	96	.99901	.00099	.00089	53.40	17
18	96514	108	.99888	.00112	.00107	52.45	18
19	96406	113	.99883	.00117	.00115	51.51	19
20	96293	115	.99881	.00119	.00119	50.57	20
21	96178	113	.99882	.00118	.00119	49.63	21
22	96065	110	.99886	.00114	.00116	48.69	22
23	95955	104	.99892	.00108	.00112	47.74	23
24	95851	98	.99898	.00102	.00105	46.80	24
25	95753	95	.99901	.00099	.00100	45.84	25
26	95658	94	.99902	.00098	.00098	44.89	26
27	95564	96	.99900	.00100	.00099	43.93	27
28	95468	99	.99896	.00104	.00102	42.98	28
29	95369	104	.99891	.00109	.00106	42.02	29
30	95265	110	.99885	.00115	.00112	41.06	30
31	95155	115	.99879	.00121	.00118	40.11	31
32	95040	122	.99872	.00128	.00125	39.16	32
33	94918	129	.99864	.00136	.00132	38.21	33
34	94789	137	.99855	.00145	.00140	37.26	34
35	94652	147	.99845	.00155	.00150	36.31	35
36	94505	158	.99833	.00167	.00161	35.37	36
37	94347	171	.99819	.00181	.00174	34.43	37
38	94176	185	.99804	.00196	.00189	33.49	38
39	93991	201	.99786	.00214	.00205	32.55	39
40	93790	220	.99765	.00235	.00224	31.62	40
41	93570	242	.99741	.00259	.00246	30.70	41
42	93328	268	.99713	.00287	.00273	29.77	42
43	93060	297	.99681	.00319	.00303	28.86	43
44	92763	330	.99644	.00356	.00337	27.95	44
45	92433	369	.99601	.00399	.00377	27.05	45
46	92064	412	.99552	.00448	.00423	26.15	46
47	91652	463	.99495	.00505	.00476	25.27	47
48	91189	520	.99430	.00570	.00538	24.40	48
49	90669	584	.99356	.00644	.00607	23.53	49
50	90085	656	.99272	.00728	.00687	22.68	50
51	89429	736	.99177	.00823	.00777	21.84	51
52	88693	825	.99070	.00930	.00878	21.02	52
53	87868	923	.98949	.01051	.00993	20.21	53
54	86945	1029	.98816	.01184	.01121	19.42	54

Age x	l_x	d_x	P_x	q_x	μ_x	$\overset{0}{e}_x$	Age x
55	85916	1144	.98669	.01331	.01263	18.65	55
56	84772	1265	.98508	.01492	.01420	17.89	56
57	83507	1393	.98332	.01668	.01590	17.16	57
58	82114	1526	.98141	.01859	.01776	16.44	58
59	80588	1664	.97935	.02065	.01978	15.74	59
60	78924	1805	.97713	.02287	.02197	15.06	60
61	77119	1947	.97475	.02525	.02433	14.40	61
62	75172	2088	.97222	.02778	.02684	13.76	62
63	73084	2228	.96951	.03049	.02953	13.14	63
64	70856	2366	.96661	.03339	.03243	12.54	64
65	68490	2499	.96352	.03648	.03553	11.95	65
66	65991	2625	.96022	.03978	.03884	11.39	66
67	63366	2745	.95668	.04332	.04239	10.84	67
68	60621	2856	.95288	.04712	.04622	10.31	68
69	57765	2959	.94878	.05122	.05036	9.79	69
70	54806	3051	.94434	.05566	.05487	9.29	70
71	51755	3130	.93953	.06047	.05976	8.81	71
72	48625	3195	.93430	.06570	.06509	8.35	72
73	45430	3243	.92861	.07139	.07092	7.90	73
74	42187	3273	.92241	.07759	.07730	7.47	74
75	38914	3282	.91566	.08434	.08432	7.05	75
76	35632	3266	.90833	.09167	.09200	6.66	76
77	32366	3225	.90037	.09963	.10042	6.28	77
78	29141	3154	.89176	.10824	.10962	5.92	78
79	25987	3054	.88248	.11752	.11964	5.57	79
80	22933	2923	.87253	.12747	.13053	5.25	80
81	20010	2763	.86192	.13808	.14231	4.94	81
82	17247	2576	.85066	.14934	.15503	4.66	82
83	14671	2365	.83878	.16122	.16863	4.39	83
84	12306	2137	.82634	.17366	.18311	4.14	84
85	10169	1897.4	.81341	.18659	.19849	3.90	85
86	8271.6	1654.1	.80003	.19997	.21468	3.68	86
87	6617.5	1414.1	.78631	.21369	.23165	3.48	87
88	5203.4	1184.6	.77235	.22765	.24928	3.30	88
89	4018.8	971.6	.75823	.24177	.26748	3.13	89
90	3047.2	779.9	.74407	.25593	.28616	2.97	90
91	2267.3	612.2	.72997	.27003	.30518	2.83	91
92	1655.1	470.0	.71604	.28396	.32439	2.70	92
93	1185.1	352.73	.70236	.29764	.34372	2.58	93
94	832.3	258.83	.68904	.31096	.36294	2.47	94
95	573.5	185.74	.67615	.32385	.38197	2.38	95
96	387.8	130.39	.66377	.33623	.40066	2.29	96
97	257.4	89.59	.65194	.34806	.41886	2.21	97
98	167.8	60.30	.64071	.35929	.43651	2.14	98
99	107.5	39.771	.63011	.36989	.45354	2.07	99
100	67.7	25.733	.62017	.37983	.46972	2.00	100
101	42.0	16.349	.61088	.38912	.48512		101
102	25.6	10.209	.60224	.39776	.49967		102
103	15.4	6.272	.59425	.40575	.51335		103
104	9.18	3.794	.58688	.41312			104
105	5.39						105

A1967/70 SELECT MORTALITY TABLE

Mortality function l

Age[x]	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	Age x+2
			34489.000	0
			34463.823	1
0	34481.408	34461.409	34440.388	2
1	34456.927	34438.320	34418.690	3
2	34433.841	34416.624	34398.727	4
3	34412.836	34397.007	34380.496	5
4	34393.221	34378.776	34363.650	6
5	34375.681	34362.274	34348.186	7
6	34359.181	34346.811	34333.760	8
7	34344.063	34332.386	34320.026	9
8	34329.638	34318.653	34306.985	10
9	34315.907	34305.612	34294.291	11
10	34303.210	34292.919	34281.602	12
11	34290.518	34280.230	34268.918	13
12	34277.830	34267.547	34255.210	14
13	34264.461	34253.497	34239.110	15
14	34250.070	34237.055	34218.225	16
15	34232.259	34215.485	34190.508	17
16	34209.439	34187.202	34154.424	18
17	34179.680	34151.017	34120.378	19
18	34143.368	34116.899	34088.257	20
19	34109.166	34084.731	34057.937	21
20	34076.957	34054.389	34029.283	22
21	34046.610	34025.734	34002.148	23
22	34017.983	33998.619	33976.374	24
23	33990.921	33972.879	33951.787	25
24	33965.254	33948.338	33928.197	26
25	33940.795	33924.799	33905.397	27
26	33917.341	33902.051	33883.161	28
27	33894.668	33879.860	33861.242	29
28	33872.531	33857.972	33839.370	30
29	33850.662	33836.106	33817.250	31
30	33829.764	33813.958	33794.559	32
31	33806.514	33791.191	33770.942	33
32	33783.557	33767.439	33746.015	34
33	33759.503	33742.299	33719.354	35
34	33733.924	33715.331	33690.498	36
35	33706.352	33686.054	33658.943	37
36	33676.272	33653.938	33624.136	38
37	33643.122	33618.409	33585.478	39
38	33606.286	33578.835	33542.311	40
39	33565.089	33534.529	33493.920	41
40	33518.794	33484.739	33439.528	42
41	33466.599	33428.646	33378.285	43
42	33407.624	33365.360	33309.271	44
43	33340.915	33293.909	33231.486	45
44	33265.431	33213.241	33143.847	46
45	33180.042	33122.213	33045.181	47
46	33083.523	33019.589	32934.221	48
47	32974.549	32904.032	32809.601	49
48	32851.686	32774.102	32669.855	50
49	32713.392	32628.250	32513.405	51
50	32558.008	32464.813	32338.568	52
51	32383.756	32282.013	32143.546	53
52	32188.740	32077.958	31926.430	54

Age[x]	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	Age x+2
53	31970.942	31850.639	31685.203	55
54	31728.226	31597.933	31417.739	56
55	31458.342	31317.610	31121.815	57
56	31158.931	31007.338	30795.116	58
57	30827.543	30664.702	30435.255	59
58	30461.645	30287.215	30039.787	60
59	30058.648	29872.344	29606.239	61
60	29615.936	29417.538	29132.138	62
61	29130.898	28920.265	28615.051	63
62	28600.975	28378.059	28052.632	64
63	28023.708	27788.571	27442.681	65
64	27396.808	27149.632	26783.206	66
65	26718.225	26459.331	26072.500	67
66	26986.236	25716.097	25309.230	68
67	25199.536	24918.797	24492.529	69
68	24357.348	24066.835	23622.102	70
69	23459.538	23160.273	22698.338	71
70	22506.732	22199.940	21722.421	72
71	21500.445	21187.559	20696.450	73
72	20443.198	20125.863	19623.545	74
73	19338.635	19018.696	18507.942	75
74	18191.617	17871.109	17355.074	76
75	17008.294	16689.418	16171.618	77
76	15796.140	15481.232	14965.496	78
77	14563.940	14255.427	13745.841	79
78	13321.717	13022.064	12522.890	80
79	12080.592	11792.241	11307.812	81
80	10852.568	10577.865	10112.467	82
			8949.0836	83
			7829.8752	84
			6766.5922	85
			5770.0459	86
			4849.6219	87
			4012.8253	88
			3264.8949	89
			2608.5274	90
			2043.7464	91
			1567.9405	92
			1176.0783	93
			861.08935	94
			614.37801	95
			426.42117	96
			287.38847	97
			187.72094	98
			118.61237	99
			72.353686	100
			42.523157	101
			24.028676	102
			13.027677	103
			6.7627284	104
			3.3540934	105
			1.5860118	106
			.71350781	107
			.30474896	108
			.12332121	109

A1967/70 SELECT MORTALITY TABLES 4% interest

Age [x]	$D_{[x]}$	$D_{[x]+1}$	D_{x+2}	Age x+2
			34489.000	0
			33138.291	1
0	34481.408	33135.970	31842.074	2
1	33131.660	31840.163	30598.090	3
2	31836.022	30596.253	29404.176	4
3	30592.886	29402.705	28258.262	5
4	29399.470	28256.848	27158.091	6
5	28254.304	27157.005	26101.798	7
6	27154.560	26100.764	25087.342	8
7	26098.665	25086.338	24112.795	9
8	25084.331	24111.830	23176.570	10
9	24109.901	23175.642	22276.917	11
10	23174.019	22276.026	21412.188	12
11	22274.466	21411.331	20581.024	13
12	21409.831	20580.201	19781.530	14
13	20578.348	19780.541	19011.763	15
14	19778.562	19010.621	18269.390	16
15	19007.958	18267.927	17552.492	17
16	18264.699	17550.795	16859.584	18
17	17546.933	16857.902	16194.979	19
18	16854.126	16193.328	15557.436	20
19	16189.657	15555.826	14945.767	21
20	15552.279	14944.210	14358.839	22
21	14940.797	14357.342	13795.567	23
22	14354.071	13794.135	13254.913	24
23	13791.012	13253.550	12735.886	25
24	13250.575	12734.592	12237.535	26
25	12731.763	12236.310	11758.953	27
26	12233.620	11757.793	11299.271	28
27	11755.233	11298.170	10857.655	29
28	11295.726	10856.607	10433.310	30
29	10854.263	10432.303	10025.471	31
30	10430.039	10024.495	9633.4073	32
31	10022.288	9632.4474	9256.4185	33
32	9630.2713	9255.4583	8893.8327	34
33	9253.2832	8892.8535	8545.0060	35
34	8890.6463	8543.9865	8209.3206	36
35	8541.7111	8208.2376	7886.1842	37
36	8205.8542	7885.0116	7575.0280	38
37	7882.4775	7573.7377	7275.3065	39
38	7571.0065	7273.8677	6986.4959	40
39	7270.8899	6984.8750	6708.0930	41
40	6981.5977	6706.2541	6439.6147	42
41	6702.6211	6437.5192	6180.5970	43
42	6433.4709	6178.2037	5930.5940	44
43	6173.6773	5927.8589	5689.1776	45
44	5922.7885	5686.0541	5455.9365	46
45	5680.3705	5452.3753	5230.4756	47
46	5446.0064	5226.4249	5012.4160	48
47	5219.2958	5007.8215	4801.3938	49
48	4999.8546	4796.1989	4597.0607	50
49	4787.3144	4591.2064	4399.0830	51
50	4581.3224	4392.5083	4207.1417	52
51	4381.5413	4199.7841	4020.9326	53
52	4187.6496	4012.7281	3840.1664	54

A1967/70 SELECT MORTALITY TABLES 4% interest

Age [x]	$D_{[x]}$	$D_{[x]+1}$	D_{x+2}	Age x+2
53	3999.3411	3831.0501	3664.5684	55
54	3816.3261	3654.4752	3493.8796	56
55	3638.3307	3482.7444	3327.8564	57
56	3465.0983	3315.6153	3166.2716	58
57	3296.3898	3152.8628	3008.9150	59
58	3131.9850	2994.2794	2855.5942	60
59	2971.6826	2839.6770	2706.1356	61
60	2815.3028	2688.8874	2560.3853	62
61	2662.6874	2541.7641	2418.2107	63
62	2513.7020	2398.1830	2279.5016	64
63	2368.2373	2258.0445	2144.1713	65
64	2226.2106	2121.2746	2012.1584	66
65	2087.5676	1987.8264	1883.4277	67
66	1952.2839	1857.6818	1757.9716	68
67	1820.3664	1730.8523	1635.8114	69
68	1691.8542	1607.3801	1516.9972	70
69	1566.8197	1487.3388	1401.6093	71
70	1445.3689	1370.8335	1289.7567	72
71	1327.6401	1257.9996	1181.5772	73
72	1213.8036	1149.0019	1077.2347	74
73	1104.0584	1044.0315	976.91702	75
74	998.62904	943.30265	880.83121	76
75	897.76009	847.04681	789.19865	77
76	801.70978	755.50684	702.24821	78
77	710.74161	668.92859	620.20821	79
78	625.11472	587.55160	543.29713	80
79	545.07270	511.59842	471.71326	81
80	470.83137	441.26304	405.62366	82
			345.15280	83
			290.37173	84
			241.28824	85
			197.83908	86
			159.88487	87
			127.20858	88
			99.518086	89
			76.453060	90
			57.596108	91
			42.487615	92
			30.643310	93
			21.573188	94
			14.800229	95
			9.8772987	96
			6.4008172	97
			4.0201797	98
			2.4424709	99
			1.4326059	100
			.80957713	101
			.43987495	102
			.22931531	103
			.11446024	104
			.05458515	105
			.02481832	106
			.01073573	107
			.00440902	108
			.00171555	109

A 1967/70 SELECT MORTALITY TABLE 4% interest

Age [x]	$N_{[x]}$	$N_{[x]+1}$	N_{x+2}	Age x+2
			835843.39	0
			801354.39	1
0	835833.48	801352.07	768216.10	2
1	801345.85	768214.19	736374.03	3
2	768208.21	736372.19	705775.94	4
3	736367.35	705774.47	676371.76	5
4	705769.82	676370.35	648113.50	6
5	676366.72	648112.41	620955.41	7
6	648108.92	620954.36	594853.61	8
7	620951.27	594852.61	569766.27	9
8	594849.63	569765.30	545653.47	10
9	569762.45	545652.55	522476.90	11
10	545650.03	522476.01	500199.99	12
11	522473.60	500199.13	478787.80	13
12	500196.81	478786.98	458206.77	14
13	478784.13	458205.79	438425.24	15
14	458202.67	438424.10	419413.48	16
15	438419.98	419412.02	401144.09	17
16	419407.09	401142.40	383591.60	18
17	401136.85	383589.92	366732.02	19
18	383584.49	366730.36	350537.04	20
19	366725.08	350535.43	334979.60	21
20	350530.32	334978.04	320033.83	22
21	334973.13	320032.34	305674.99	23
22	320027.63	305673.56	291879.43	24
23	305669.08	291878.06	278624.51	25
24	291873.80	278623.22	265888.63	26
25	278619.17	265887.40	253651.09	27
26	265883.55	253649.93	241892.14	28
27	253646.27	241891.04	230592.87	29
28	241887.55	230591.82	219735.21	30
29	230588.47	219734.21	209301.91	31
30	219730.97	209300.93	199276.43	32
31	209297.76	199275.47	189643.03	33
32	199272.34	189642.07	180386.61	34
33	189638.91	180385.63	171492.78	35
34	180382.40	171491.76	162947.77	36
35	171488.40	162946.69	154738.45	37
36	162943.13	154737.28	146852.27	38
37	154733.45	146850.97	139277.24	39
38	146846.80	139275.80	132001.93	40
39	139271.20	132000.31	125015.43	41
40	131995.19	125013.60	118307.34	42
41	125007.87	118305.25	111867.73	43
42	118298.80	111865.33	105687.13	44
43	111858.07	105684.39	99756.536	45
44	105676.20	99753.413	94067.358	46
45	99744.168	94063.797	88611.422	47
46	94053.378	88607.371	83380.946	48
47	88595.648	83376.352	78368.530	49
48	83363.190	78363.335	73567.136	50
49	78348.597	73561.282	68970.076	51
50	73544.823	68963.501	64570.993	52
51	68945.176	64563.635	60363.851	53
52	64543.296	60355.647	56342.918	54

A 1967/70 SELECT MORTALITY TABLE 4% interest

Age [x]	$N_{[x]}$	$N_{[x]+1}$	N_{x+2}	Age x+2
53	60333.143	56333.802	52502.752	55
54	56308.985	52492.659	48838.184	56
55	52465.379	48827.049	45344.304	57
56	48797.161	45332.063	42016.448	58
57	45299.429	42003.039	38850.176	59
58	41967.525	38835.540	35841.261	60
59	38797.026	35825.344	32985.667	61
60	35783.721	32968.419	30279.531	62
61	32923.597	30260.910	27719.146	63
62	30212.820	27699.118	25300.935	64
63	27647.715	25279.478	23021.434	65
64	25224.747	22998.537	20877.262	66
65	22940.498	20852.930	18865.104	67
66	20791.642	18839.358	16981.68	68
67	18774.923	16954.557	15223.705	69
68	16887.127	15195.273	13587.893	70
69	15125.055	13558.235	12070.896	71
70	13485.489	12040.120	10669.287	72
71	11965.170	10637.530	9379.5300	73
72	10560.758	9346.9547	8197.9528	74
73	9268.8080	8164.7496	7120.7181	75
74	8085.7327	7087.1037	6143.8011	76
75	7007.7767	6110.0167	5262.9698	77
76	6030.9878	5229.2780	4473.7712	78
77	5151.1932	4440.4516	3771.5230	79
78	4363.9811	3738.8664	3151.3148	80
79	3664.6888	3119.6161	2608.0176	81
80	3048.3988	2577.5674	2136.3044	82
			1730.6807	83
			1385.5279	84
			1095.1562	85
			853.86794	86
			656.02886	87
			496.14399	88
			368.93541	89
			269.41733	90
			192.96427	91
			135.36816	92
			92.880543	93
			62.237233	94
			40.664045	95
			25.863816	96
			15.986518	97
			9.5857004	98
			5.5655208	99
			3.1230498	100
			1.6904439	101
			.88086681	102
			.44099186	103
			.21167655	104
			.09721631	105
			.04263116	106
			.01781284	107
			.00707711	108
			.00266810	109

End of Appendix.