MTH4104 - INTRODUCTION TO ALGEBRA - 2019/20

YOU CAN PREVIEW THIS QUIZ, BUT IF THIS WERE A REAL ATTEMPT, YOU WOULD BE BLOCKED BECAUSE:

This quiz is not currently available

QUESTION 1 Not yet answered Marked out of 10.00

Let R be the relation $\{(x, y) \in \mathbb{R}^2 : y > x + 2\}$ on the set $\mathbb{R}$ of real numbers. True or false:

R is reflexive.

R is symmetric.

R is transitive.

QUESTION 2

Not yet answered Marked out of 10.00

Let R be a ring with identity, and let $x \in R$. Then the equation $(-1)x = -x$ is true in R.

Fill in the white boxes to make a proof of this fact. For each step in the proof, fill in the grey box to indicate which ring axiom or other fact is used to derive that step.

Proof:

- By [ ], we have [ ].
- By [ ], we have [ ].
- By [ ], we have [ ].
- By [ ], we have [ ].
- By [ ], we have $(-1)x = -x$.

(You should fill all the boxes, so if you came up with a proof that doesn't fill them all, try including more intermediate steps. Please feel free to look up in the lecture notes what "Proposition 5.11" is.)

uniqueness of additive inverses | the distributive law

the multiplicative identity law | the additive commutative law

the additive identity law | the additive inverse law

Proposition 5.11

$(-1 + 1)x = 0$ | $(-1)x + 1x = 0$ | $0((-x) + x) = 0$ | $0x = 0$ | $(-1)(-x) = 1x$ | $(-1)x + x = 0$

QUESTION 3

Not yet answered Marked out of 10.00

Let X be the set $\{a + bi : a, b \in \{1, ..., 8\}\}$. That is, $X = \{ 1+i, 1+2i, ..., 1+8i, 2+i, ..., 8+8i \}$.

Let R be the relation $\{(x, y) \in X^2 : |x| = |y|\}$. Here $|\ |$ means the complex modulus, $|a + bi| = \sqrt{a^2 + b^2}$. You may assume that R is an equivalence relation.

Write down the equivalence class $[1+7i]_R$. Write the elements in increasing order of their real part (e.g. if you get the answer $\{3+i, 2+4i\}$, you should enter $\{$**2**$+4i$, **3**$+i\}$.)

Answer:

QUESTION 4

Not yet answered Marked out of 10.00

Calculate $[43]_{140}^{-1}$, and express it in the form $[b]_{140}$ where $0 \leq b < 140$. What is b?

Answer:

QUESTION 5

Not yet answered Marked out of 10.00

Recall the Division Rule for Polynomials (Theorem 2.12) for real coefficients:

> *Let f and g be two polynomials in $\mathbb{R}[x]$, with $g \neq 0$. Then there exist polynomials q and r in $\mathbb{R}[x]$ such that $f = qg + r$, and either $r = 0$ or $\deg r < \deg g$.*

We proved this by induction. In the inductive case of the proof, we applied the inductive hypothesis to two polynomials we called f^* and g, to produce two polynomials q^* and r^*.

Suppose $f = 21x^3 + 59x^2 + 23x + 40$ and $g = 3x + 8$. Find f^*. Enter the leading coefficient of f^* (that is, the coefficient of the nonzero term with greatest exponent).

Answer:

QUESTION 6

Not yet answered Marked out of 10.00

The statement below is the Fundamental Theorem of Arithmetic with the condition that $p_1, \ldots, p_k$ must be *different* primes deleted. This statement is false.

> *Every positive integer n can be written as a product $n = p_1^{e_1} \cdot \ldots \cdot p_k^{e_k}$ where $p_1, \ldots, p_k$ are prime numbers and $e_1, \ldots, e_k$ are positive integers. This expression is unique up to reordering of the factors $p_i^{e_i}$.*

Why is the statement false? Choose the option that is true and contradicts the above statement.

Select one:

- There is an expression $p_1^{e_1} \cdot \ldots \cdot p_k^{e_k}$ of this kind which is not a factorisation of any positive integer n.
- There is a positive integer n with more than one factorisation of this kind which the statement counts as different.
- There is a positive integer n with no factorisation of this kind.
- There are two expressions $p_1^{e_1} \cdot \ldots \cdot p_k^{e_k}$ of this kind which the statement counts as the same, but are factorisations of different positive integers n.

QUESTION 7

Not yet answered Marked out of 10.00

Let n be a positive integer such that $\gcd(n, 2^5 \cdot 3^2 \cdot 5) = 2^3 \cdot 3^2$. Suppose the prime factorisation of n is $n = 2^e \cdot 3^f \cdot 5^g \cdot \ldots$. What are the possible values that e and f may have?

The possible values of e are:

The possible values of f are:

QUESTION 8

Not yet answered Marked out of 10.00

Let

$$A = \begin{pmatrix} [3]_7 & [0]_7 \\ [4]_7 & [1]_7 \end{pmatrix} \text{ and } B = \begin{pmatrix} [3]_7 & [5]_7 \\ [5]_7 & [3]_7 \end{pmatrix}$$

be matrices in $M_2(\mathbb{Z}_7)$. Compute their product AB, and express the top right entry in the form $[a]_7$, where $0 \le a < 7$. What is a?

Answer:

QUESTION 9

Not yet answered Marked out of 10.00

Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function $f(n) = (n^2 + n)/2$.

Suppose we try to define a function $F : \mathbb{Z}_m \to \mathbb{Z}_m$ by the definition $F([a]_m) = [f(a)]_m$ for any integer a. Whether F is well-defined depends on the value of m. In fact, F is well-defined for one of the two values $m = 7$ and $m = 8$, and not well-defined for the other. For which one is F well defined?

Select one:

- 8
- 7

QUESTION 10

Not yet answered Marked out of 10.00

What is the multiplicity of $x = -8$ as a solution to the equation $x^4 + 18x^3 + 93x^2 + 80x - 192 = 0$?

Answer: