

MTH4100: Calculus I

Duration: 2 hours

Date and time: 15 December 2016, 10:00h–12:00h

Student ID:

Name:

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You should attempt ALL questions. Marks awarded are shown next to the questions. Write your calculations and answers in the boxes provided and perform additional calculations on the blank pages at the end of the booklet.

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Examiner(s): W. Just

Question 1. Find the natural domain and the range of the function

[8]

$$f(x) = \sqrt{\frac{1-x}{x}}.$$

Question 2. Write a formula for $f \circ g \circ h$ where $f(x) = \cos(x)$, $g(x) = x + 1$, and $h(x) = \sqrt{x}$. [8]

Question 3. Find the limit

[8]

$$\lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x + 2} - 2}.$$

Question 4. At what points is the following function continuous?

[8]

$$f(x) = \begin{cases} 5 - x^2, & x \leq 0 \\ |\sin(x) - 5|, & x > 0 \end{cases}$$

Question 5. Compute the derivative of the following function

[8]

$$f(x) = \ln(1 + \sqrt{x}).$$

Question 6. Let the function f be defined by

[8]

$$f(x) = 2x + \ln(x - 2), \quad x \geq 2.$$

Find the value of the derivative of the inverse, $(f^{-1})'(x)$, at $x = f(3) = 6$.

Question 7. Find the absolute maximum and minimum values of the function

[8]

$$f(x) = x\sqrt{5 - x^2}$$

on the interval $[-2, 1]$.

Question 8. Write down the following finite sum using Σ notation

[8]

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15}.$$

Question 9. Find the derivative of y with respect to x if

[8]

$$y = \int_0^{\ln|x|} t \sin(e^t) dt.$$

Question 10. Evaluate the integral

[10]

$$\int_0^2 \frac{1}{x^2 + 2x + 2} dx.$$

Question 11. Evaluate the following indefinite integral

[8]

$$\int x^2 e^{-3x} dx.$$

Question 12. Evaluate the improper integral

[10]

$$\int_1^{\infty} \frac{1}{(3+x)\sqrt{x}} dx.$$

End of Paper.

MTH4200: Calculus I

Duration: 2 hours

Date and time: 15 December 2016, 14:30h–16:30h

Student ID:

Name:

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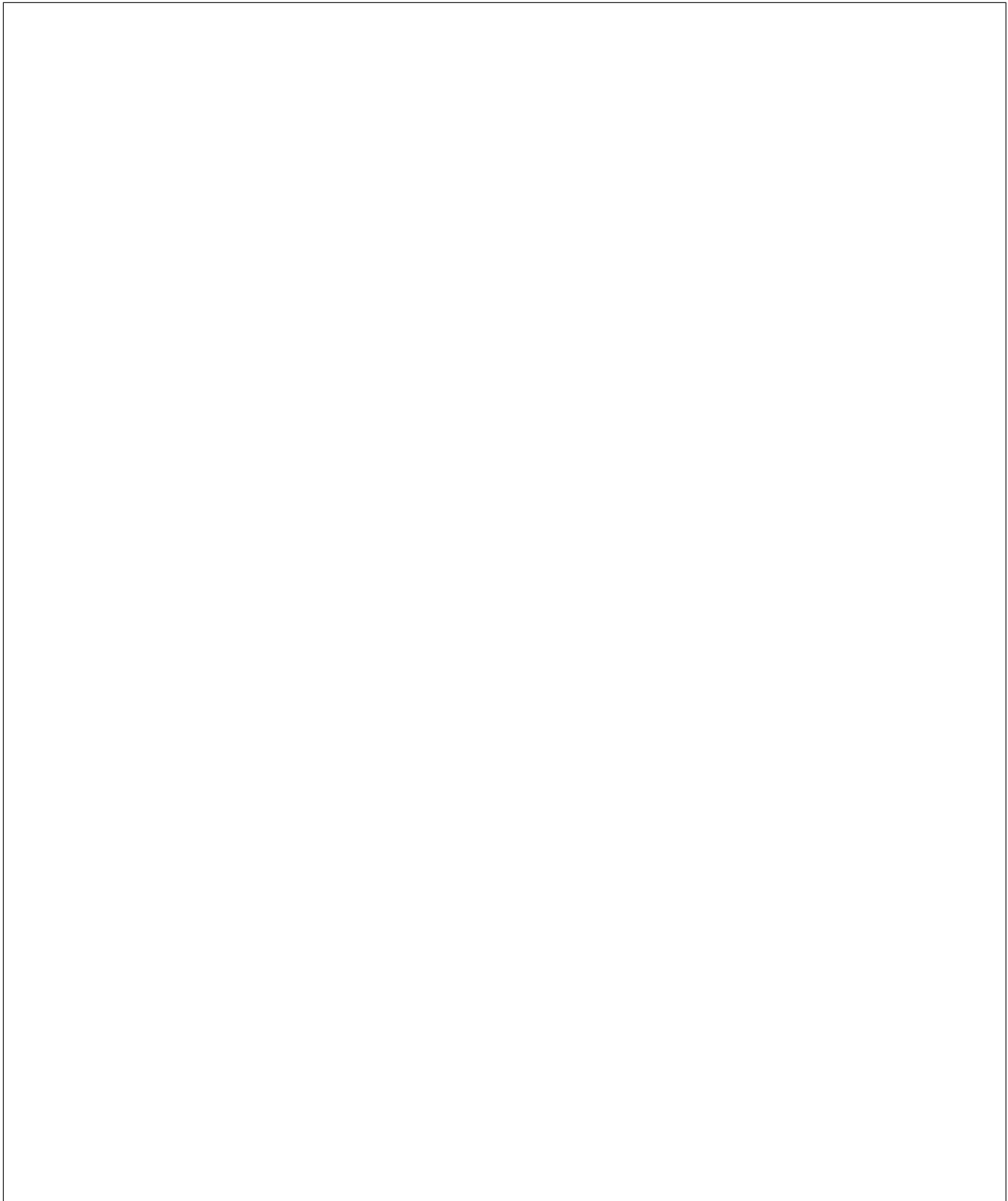
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Examiner(s): W. Just

Question 1. Graph the function

[8]

$$f(x) = \begin{cases} e^x, & x < 0 \\ \ln(x+1), & x \geq 0 \end{cases} .$$



Question 2. Compute all values of θ which satisfy the equation

[8]

$$\cos(2\theta) + \sin(\theta) = 1.$$

Question 3. Show that the equation

[8]

$$x^6 - 3x^4 + x^3 - 2 = 0$$

has two solutions in the interval $[-2, 2]$.

Question 4. Compute the limit

[8]

$$\lim_{x \rightarrow \infty} \frac{3x^4 - x^2 + 1}{-2x^4 - x^3 + x}.$$

Question 5. Compute the derivative of the following function

[8]

$$f(x) = \cos(x + e^{-x}) .$$

Question 6. Determine the linearisation of the function

[8]

$$f(x) = (x + 1)^3 \sqrt{x^2 + 5}$$

at $x = -2$.

Question 7. Determine all critical points for the following function

[8]

$$f(x) = x^2(x + 1)^{1/5}.$$

Question 8. On what open intervals is f increasing or decreasing?

[8]

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + 1$$

Question 9. Calculate the following indefinite integral

[8]

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx .$$

Question 10. Calculate the definite integral

[10]

$$\int_0^2 |x^2 - 2| dx.$$

Question 11. Compute the following limit

[8]

$$\lim_{x \rightarrow 0} \frac{\operatorname{arsinh}(x)}{x}.$$

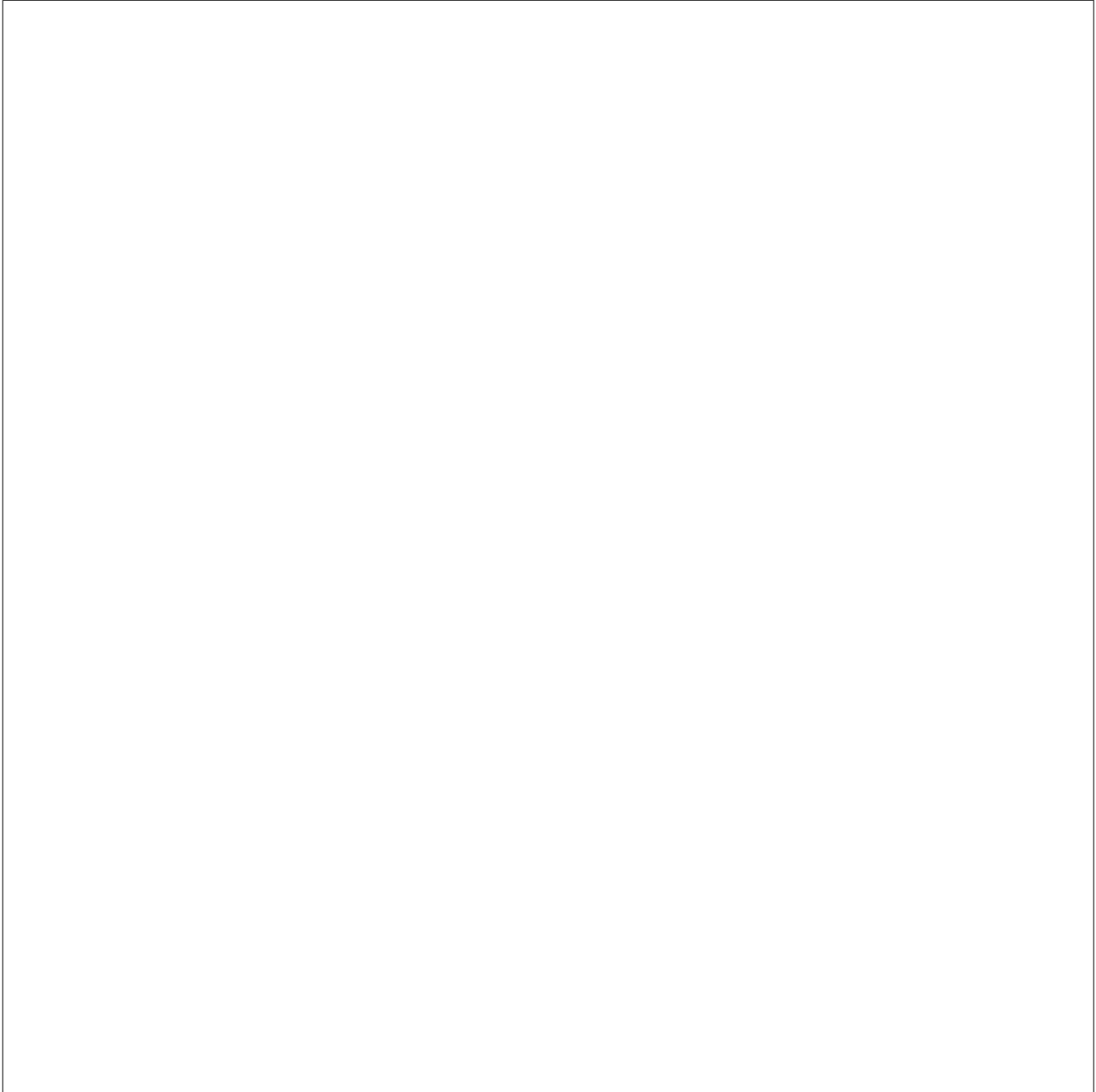
(Here $\operatorname{arsinh} = \sinh^{-1}$ denotes the inverse function of the hyperbolic sine.)

Question 12. Perform a test to determine if the integral

[10]

$$\int_0^{\infty} \frac{e^{-x}}{\sqrt{x}} dx$$

converges or diverges.



End of Paper.