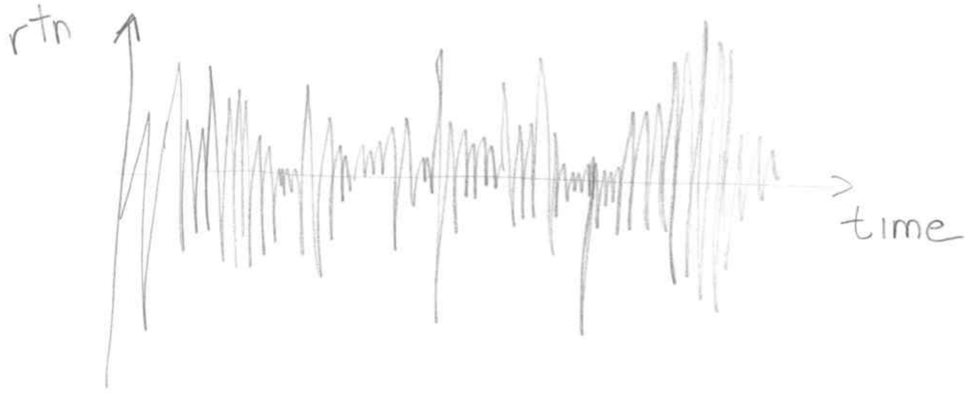
CLASS 8: CONDITIONAL HETEROSCEDASTIC MODELS

- IN THE CONTEXT OF FINANCIAL TIME SERIES THE VARIANCE OF THE ERROR TERM MIGHT BE NON-CONSTANT OVER TIME; SO IT IS INTERESTING TO DESCRIBE HOW IT EVOLVES OVER TIME.

- ANOTHER IMPORTANT CHARACTERISTIC OF FINANCIAL RETURNS IS KNOWN AS VOLATILITY CLUSTERING

VOLATILITY CLUSTERING DESCRIBES THE TENDENCY OF LARGE CHANGES IN ASSET PRICES TO FOLLOW LARGE CHANGES AND SMALL CHANGES TO FOLLOW SMALL CHANGES



IN OTHER WORDS, THE ACTUAL LEVEL OF VOLATILITY TENDS TO BE CORRELATED TO ITS LEVEL IN THE PERIODS, WHICH IMMEDIATELY PROCEED

ONE WAY TO STUDY THIS PHENOMENON IS TO USE CONDITIONAL HETEROSKEDASTIC MODELS: ARCH AND GARCH MODELS

ARCH MODEL

THE AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTIC MODEL IS SPECIFIED AS:

$$Y_t = c + \phi_1 Y_{t-1} + \ldots\ldots + e_t \qquad e_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \ldots + \alpha_q e_{t-q}^2$$

THIS MODEL IS REFERRED TO AS ARCH (q) WHERE q REFERES TO THE ORDER OF THE LAGGED SQUARED ERRORS.

A SPECIAL CASE OF THIS MODEL IS ARCH (1) GIVEN BY:

$$Y_t = c + \phi_1 Y_{t-1} + \ldots + e_t \qquad e_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2$$

TO MAKE SURE THAT THIS CLASS OF MODELS IS APPROPRIATE FOR THE DATA, WE NEED TO TEST WHETHER ARCH-EFFECTS ARE PRESENT IN THE RESIDUALS OF THE ESTIMATED MODEL.

ARCH TEST:

H_0: NO ARCH EFFECTS

H_1: ARCH EFFECTS

- AUTOCORRELATION OF SQUARED RETURNS

AUTOCORRELATION OF SQUARED RETURNS REFLECTS THE VOLATILITY CLUSTERING CHARACTERISTICALLY OBSERVED IN RETURNS AND IS EVIDENCE OF ARCH.

THIS CAN BE TESTED BY COMPUTING BOTH ACF AND THE LJUNG-BOX STATISTIC USING SQUARED RETURNS

• FORMAL ARCH TEST

CONSIDERING AN ARCH(1) MODEL, A TEST OF ARCH IS GIVEN BY TESTING THAT $\alpha_1 = 0$.

IF THIS HYPOTHESIS IS NOT REJECTED, THE MODEL UNDER H_0 REDUCES A NORMAL DISTRIBUTION WITH ZERO MEAN AND CONSTANT VARIANCE.

$$H_0 : \alpha_1 = 0$$
$$H_1 : \alpha_1 \neq 0$$

CONSIDERING AN ARCH(q) MODEL, WE HAVE:

$$Y_t = c + \phi Y_{t-1} + \dots + e_t \qquad e_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \dots + \alpha_q e_{t-2}^2$$

ARCH TEST:

$$H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_q = 0 \qquad \text{(NO ARCH)}$$
$$H_1 : \alpha_1 \neq 0 \text{ OR } \dots \text{ OR } \alpha_q \neq 0 \qquad \text{(ARCH)}$$

GARCH MODEL

THE DRAWBACK OF ARCH MODEL IS THAT OFTEN IT REQUIRES MANY PARAMETERS.

A WAY TO SOLVE THIS PROBLEM IS TO SPECIFY THE CONDITIONAL VARIANCE ALSO AS A FUNCTION OF ITS OWN LAGS.

THE GARCH (GENERALISED ARCH) MODEL IS DEFINED AS:

$$y_t = c + \phi_1 y_{t-1} + \dots + e_t \qquad e_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \dots + \alpha_q e_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_p \sigma_{t-p}^2$$

THIS MODEL IS REFERRED TO AS GARCH (p, q)

A SPECIAL CASE IS GARCH (1, 1) MODEL

$$y_t = c + \phi_1 y_{t-1} + \dots + e_t \qquad e_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

- THE ADVANTAGE OF USING THE GARCH MODEL, COMPARED TO THE ARCH, IS THAT IT IS MORE PARSIMONIOUS IN THE ESTIMATION OF THE PARAMETERS.
  THEREFORE, IT IS LESS LIKELY THAT WE BREAK THE NON-NEGATIVITY CONSTRAINTS

- MORE SPECIFICALLY, THE GARCH (1,1) MODEL CAN BE RE-WRITTEN AS AN INFINITE ORDER ARCH MODEL.

Exercise 8.1

BUILD THE MODEL FOR 3M STOCK RETURNS

a) PLOT RETURNS SERIES

b) SELECT THE APPROPRIATE MODEL FOR OUR DATA

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WE USE CORRELOGRAM :

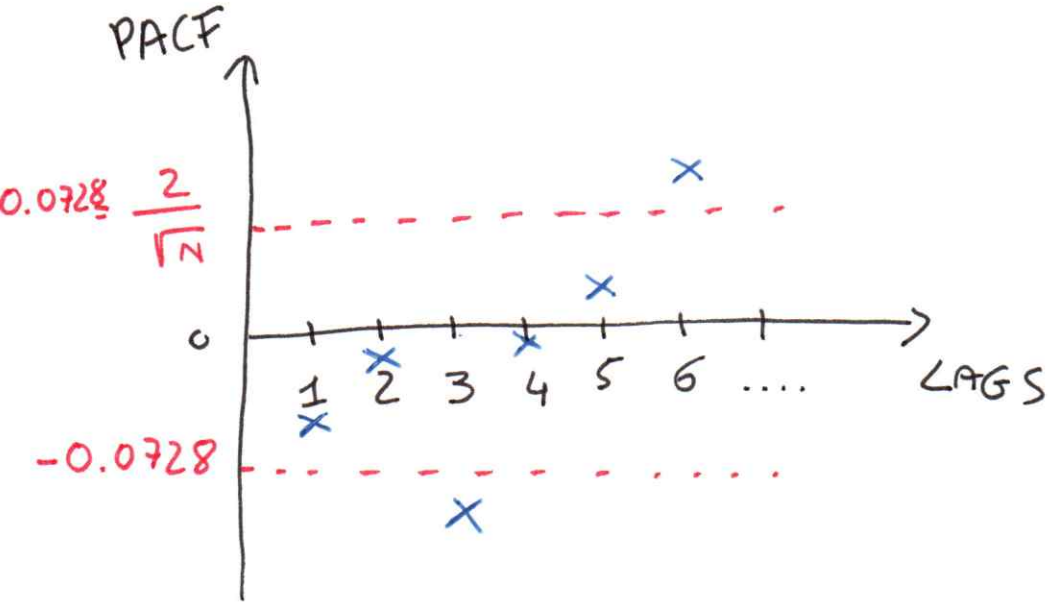
PACF → IT IS COMPUTED TO IDENTIFY THE BEST ORDER OF AR MODEL

ACF → IT IS COMPUTED TO IDENTIFY THE BEST ORDER OF MA MODEL

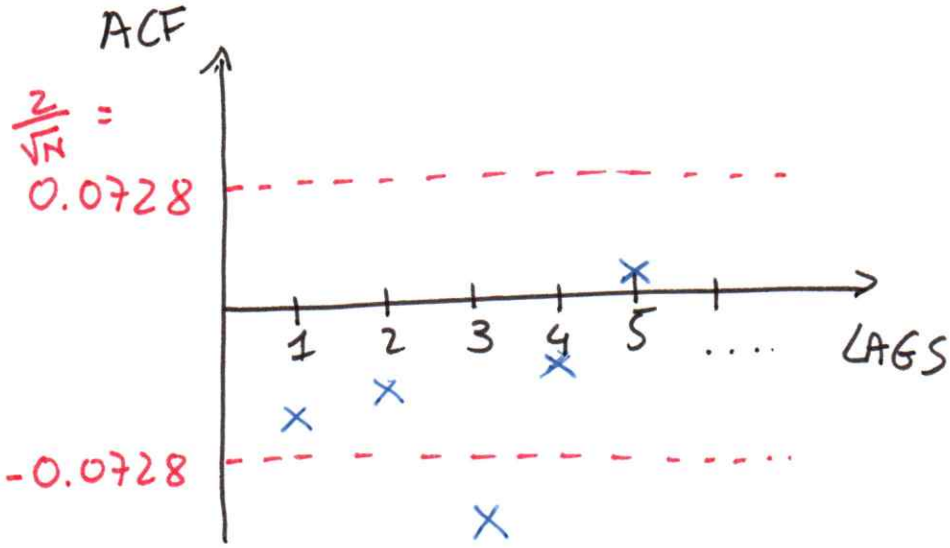
WE NEED TO TEST THE SIGNIFICANCE OF CORRELATION ρ_K

- CORRELATION AT LAG K IS SIGNIFICANT IF $|\hat{\rho}_K| > \frac{2}{\sqrt{N}}$ where N = n° obs
- IF $|\hat{\rho}_K| \leq \frac{2}{\sqrt{N}}$ THEN CORRELATION AT LAG K IS NOT SIGNIFICANTLY DIFFERENT FROM ZERO

IN OUR CASE: $\frac{2}{\sqrt{N}} = \frac{2}{\sqrt{755}} = 0.0728$



$\boxed{AR(3)}$



$\boxed{MA(3)}$

WE CAN ESTIMATE AN AR(3) MODEL

c) CHECK FOR ARCH EFFECTS

TWO WAYS ON HOW TO TEST ARCH EFFECTS:

- autocorrelation of squared returns

  - WE GENERATE A NEW SERIES : r_t^2
  - CORRELOGRAM
  - ACF: CORRELATION COEFFICIENTS ARE STATISTICALLY SIGNIFICANT
  - LJUNG-BOX STATISTIC (Q-STAT)

H_0 : NO CORR.
H_1 : CORR.
} THIS IS A JOINT MULTIPLE HYPOTHESIS

P_VALUE $< \alpha$ $\rightarrow$ CORRELATION

$\downarrow$

WE OBSERVE AUTOCORRELATION IN SQUARED RETURNS. THIS REFLECTS VOLATILY CLUSTERING AND IT IS EVIDENCE OF ARCH EFFECTS

- ARCH test

WE CONSIDER AN ARCH (1) MODEL

$$r_t = c + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \phi_3 r_{t-3} + e_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2$$

OR

$$r_t = c + \phi_3 r_{t-3} + e_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2$$

→ ARCH TEST

THIS IS A TEST FOR HETEROSKEDASTICITY

$H_0: \alpha_1 = 0$ (NO ARCH EFFECTS)

$H_1: \alpha_1 \neq 0$ (ARCH EFFECTS)

IN OUR CASE:

P_VALUE $< \alpha$

WE REJECT H_0, SO THERE IS A PRESENCE OF ARCH EFFECTS

d) GARCH MODEL

WE TEST A GARCH (1, 1) MODEL

$$r_t = c + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \phi_3 r_{t-3} + e_t \qquad e_t = \varepsilon_t \sigma_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

THE ESTIMATION OUTPUT SHOWS THAT PARAMETERS ϕ_1 AND ϕ_2 ARE NOT SIGNIFICANT. WE DROP THEM AND WE ESTIMATE THE MODEL:

$$r_t = c + \phi_3 r_{t-3} + e_t \qquad e_t = \varepsilon_t \sigma_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

WE OBTAIN:

$$r_t = 0.01 - 0.08\, r_{t-3} + e_t$$

$$\sigma_t^2 = 0.000562 + \underline{0.075}\, e_{t-1}^2 + \underline{0.785}\, \sigma_{t-1}^2$$

THIS SUM $(\alpha_1 + \beta_1) < 1$

SO THE SHOCKS TO THE CONDITIONAL VARIANCE ARE HIGHLY PERSISTENT

indicator of persistence

e) RESIDUAL CHECK

ARE THE RESIDUAL i.i.d?

WE TEST IT BY USING THE CORRELOGRAM (ACF) OF THE RESIDUALS

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THERE IS NO CORRELATION

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RESIDUALS ARE A WHITE NOISE PROCESS

d) FORECAST

WE FORECAST THE LAST YEAR (2008) WITH A ONE - STEP AHEAD FORECAST

IN EVIEWS WE HAVE TWO OPTIONS:

- DYNAMIC FORECAST
- STATIC FORECAST

- DYNAMIC CALCULATES DYNAMIC, MULTI-STEP FORECASTS STARTING FROM THE FIRST PERIOD IN THE FORECAST SAMPLE. IN DYNAMIC FORECASTING, PREVIOUSLY FORECASTED VALUES FOR THE LAGGED DEPENDENT VARIABLES ARE USED IN FORMING FORECASTS OF THE CURRENT VALUE

- STATIC CALCULATES A SEQUENCE OF ONE-STEP AHEAD FORECASTS, USING THE ACTUAL, RATHER THAN FORECASTED VALUES FOR LAGGED DEPENDENT VARIABLES, IF AVAILABLE.