

## Quiz 5: mini problems 1, 2, 3

### Question 1.

Suppose  $X_1, \dots, X_t$  is a sample from a stationary MA(1) time series

$$X_t = 0.2X_{t-1} + \varepsilon_t,$$

where  $\varepsilon_t$  is an i.i.d. sequence with zero mean and variance 1.

- (a) Find the 1-step ahead forecast  $\hat{X}_t(1)$  of  $X_{t+1}$ , the forecast error and the variance of the forecast error.
- (b) Find the 2-step ahead forecast  $\hat{X}_t(2)$  of  $X_{t+2}$ , the forecast error and the variance of the forecast error.
- (c) What can you say about the  $k$ -step ahead forecast  $\hat{X}_t(1)$  this time series?

### Question 2.

Suppose  $X_1, \dots, X_t$  is a sample from a stationary MA(1) time series

$$X_t = 1 + \varepsilon_t - 0.8\varepsilon_{t-1},$$

where  $\varepsilon_t$  is an i.i.d. sequence with zero mean and variance 1.

- (a) Find the 1-step ahead forecast  $\hat{X}_t(1)$  of  $X_{t+1}$ , the forecast error and the variance of the forecast error.
- (b) Find the 2-step ahead forecast  $\hat{X}_t(2)$  of  $X_{t+2}$ , the forecast error and the variance of the forecast error.
- (c) What can you say about the  $k$ -step ahead forecast  $\hat{X}_t(1)$  this time series?

**Question 3.**

Consider an AR(1) time series

$$X_t = 1 - 0.4X_{t-1} + \varepsilon_t,$$

where  $\varepsilon_t$  is a white noise sequence with zero mean and variance  $\sigma_\varepsilon^2$ .

- Find  $E[X_t]$ .
- Given  $X_t = 5$ , find the 1-step ahead forecast  $\hat{X}_t(1)$  of  $X_{t+1}$