ECOM073: Topics in Financial Econometrics

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http://faculty.chicagogsb.edu/ruey.tsay/teaching/fts2/
here you can find various data sets.

Exercise 2.

2.1. In **d-ibm3dx7008.txt** you will find the daily simple stock returns of IBM for the period 1970.01.02 to 2008.12.31.

(a) compute the histogram, sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of the returns of this stock.

(b) Discuss the summary characteristics of these returns. In particular, conduct test to clarity the following questions and comment on your findings:

(i) is the mean significant?
(ii) is the skewness equal to 0?
(iii) is the excess kurtosis equal to 0?
(iv) is the distribution of the returns normal? (Use Jargue-Bera test).

(c) Together with the histogram plot the probability density of the normal distribution which mean and standard deviation are equal to the sample mean and the sample standard deviation of your data. Comment on your finding.

(d) Assuming that the given returns are log returns, compute the aggregated return for the period 1970.01.02 to 2008.12.31. Comment on your finding.

2.2. In d-3stock.txt you will find the daily stock returns of American Express (axp), Caterpilar (cat) and Starbucks (sbux) for the period 1994.01.03 to 2003.12.32.

(a) Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of the returns of each stock.

(b) Discuss the summary characteristics characteristics of these returns. In particular, focuss on the following questions:
(i) is the mean significant?
(ii) is the skewness equal to 0?
(iii) is the excess kurtosis equal to 0?

Based on these characteristics what can you say about differences/ similarities between American Express (axp), Caterpilar (cat) and Starbucks (sbux) daily stock returns?
Are these stock returns normally distributed?

(c) Assuming the returns are log returns, compute the aggregated return for the period 1994.01.03 to 2003.12.32.

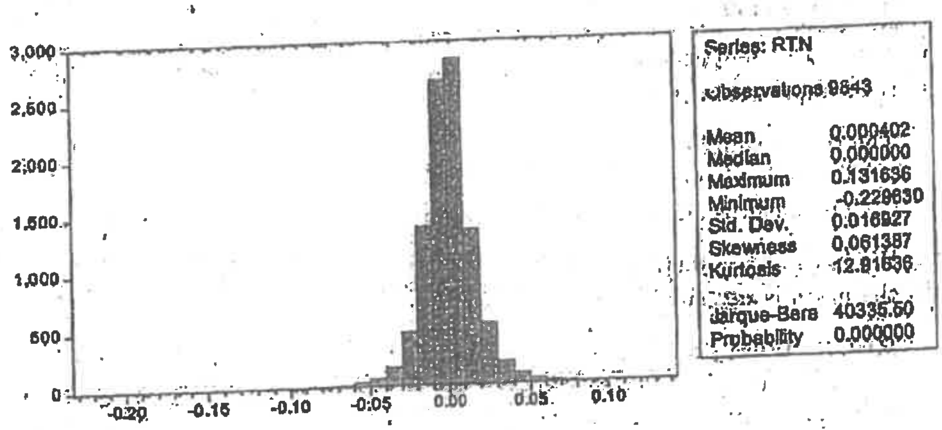
2.3. Using summary statistics from table 1.2, test for significance of the mean, skewness and access kurtosis for the Microsoft daily and monthly log-returns.
Are these stock returns normally distributed? Comment on your findings.

TABLE 1.2 Descriptive Statistics for Daily and Monthly Simple and Log Returns of Selected Indexes and Stocks[a]

Security	Start	Size	Mean	Standard Deviation	Skewness	Excess Kurtosis	Minimum	Maximum
				Daily Log Returns (%)				
SP	70/01/02	9845	0.023	1.062	−1.17	30.20	−22.90	10.96
VW	70/01/02	9845	0.035	1.008	−0.94	21.56	−18.80	10.90
EW	70/01/02	9845	0.072	0.816	−1.00	17.76	−10.97	10.20
IBM	70/01/02	9845	0.026	1.694	−0.27	12.17	−26.09	12.37
Intel	72/12/15	9096	0.066	2.905	−0.54	7.81	−35.06	23.41
3M	70/01/02	9845	0.034	1.488	−0.78	20.57	−30.08	10.92
Microsoft	86/03/14	5752	0.095	2.369	−0.63	14.23	−35.83	17.87
Citi-Grp.	86/10/30	5592	0.033	2.575	0.22	33.19	−30.66	45.63
				Monthly Log Returns (%)				
SP	26/01	996	0.43	5.54	−0.52	7.93	−35.58	35.22
VW	26/01	996	0.74	5.43	−0.58	6.85	−34.22	32.47
EW	26/01	996	0.96	7.14	0.25	8.55	−37.51	51.04
IBM	26/01	996	1.09	7.03	−0.07	2.62	−30.37	38.57
Intel	73/01	432	1.39	12.80	−0.55	3.06	−59.54	48.55
3M	46/02	755	1.03	6.37	−0.08	1.25	−32.61	22.95
Microsoft	86/04	273	2.01	10.66	0.10	1.59	−42.09	41.58
Citi-Grp	86/11	266	0.68	10.09	−1.09	3.76	−49.87	23.18

Solution of Exercise 2.1. (a) The time series of returns $X_1, \cdots, X_N$ you analyze, is a stationary sequence. That means, each of these observations X_j has the same distribution and the same probability density f.

- We can visualize the probability density f using the histogram, which shows the shape of unknown probability density.
- Below see the output of histogram and the summary characteristics.



Summary characteristics show that the mean is close to zero, standard deviation is small, skewness is almost zero, but kurtosis 12.91 is much higher that kurtosis of the normal distribution which is 3. This is indication that the distribution of these returns may be not normal.

Next we do the following rigorous testing for significance of the mean, skewness and access kurtosis.

(b) 1. **Testing for significance of the mean.** First we test if the mean EX of returns is significant or not. "Significant" means that $EX \neq 0$ (i.e. significantly different from zero). Not "significant" means $EX = 0$.

Note: the sample mean $\bar{X} = 0.0004302$ is close to 0, and we expect it to be close to EX. It shows that very likely the mean is 0, but we need to test that formally.

Formally, we need to test the hypothesis:

H_0: $EX = 0$ (mean not significant) against alternative H_1: $EX \neq 0$ (mean is significant).

To test this hypothesis at 5% significance level we use the rule which involves the sample mean $\bar{X} = 0.0004302$, and the standard deviation $SD = 0.0169$.

The rule is simple:

The mean is not significant at 5% level if $|\bar{X}| \leq 2SD/\sqrt{n}$.

The mean is significant at 5% level if $|\bar{X}| > 2SD/\sqrt{n}$, n - sample size.

We have $|\bar{X}| = 0.0004302 > 2SD/\sqrt{n} = 2(0.0169)/\sqrt{9843} = 0.00034$, which mean that the mean is significantly different from 0, i.e. $EX \neq 0$.

Recall: What does it mean "a test with significance level $\alpha = 5\%$"? Such test will make exactly 5% mistakes when the null hypothesis H_0 is true, i.e. in 5 cases out of 100 when data has 0 mean, the test would find that the mean is not zero and reject the true H_0 hypothesis.

How we obtain such rejection rule? We construct the test statistics (t-ratio statistic)

$$t = \frac{\sqrt{n}\bar{X}}{SD}, \qquad note: \qquad \sqrt{n}\bar{X} \sim N(0, SD^2).$$

If null hypothesis is true, i.e. $EX = 0$, and the number of observations is large, it has normal distribution:

$$t \sim N(0, 1).$$

According to statistical theory, we reject H_0 at 5% significance level, if

$$|t| \geq z_{2.5\%} \sim 2.$$

(Here $z_{2.5\%}$ is 2.5% upper percentile of the standard normal distribution). Then $|t| = \sqrt{n}|\bar{X}|/SD \geq 2$ implies the rule we used above to H_0:

Rule: If $|\bar{X}| > 2SD/\sqrt{n}$, then μ is significantly different from zero ($\mu \neq 0$).

2. Testing for skewness. Next we test if the skewness $S(X)$ of the distribution of returns is significantly different from 0 or not.

Note: the sample skewness $\hat{S}(X) = 0.061387$ is close to 0. The zero skewness $S(X) = 0$ means that probability density is symmetric around the mean.

Indeed, the histogram suggests that the density is symmetric around the zero mean. This indicates that very likely the skewness $S(X)$ is 0, but we need to test that formally. We have $\hat{S}(X) = 0.061387$ and the number of observations $N = 9843$

We test the hypothesis at 5% significance level:

H_0: $S(X) = 0$ against alternative H_1: $S(X) \neq 0$.

We construct the test statistics:

$$t = \frac{\hat{S}(X)}{\sqrt{6/N}} = \frac{0.061387}{\sqrt{6/9843}} = 2.4864.$$

Under the null hypothesis, $t \sim N(0, 1)$ is normally distributed. Therefore the testing rule is similar to that we used for testing for significance of the mean:

Rule: we reject H_0 at 5% significance level, if

$$|t| \geq z_{2.5\%} \sim 2.$$

In our case $t = 2.4864 > 2$. Hence, the test shows that there is enough evidence in the data to reject the null hypothesis of zero skewness $S(X) = 0$.

Comment: The number of observations $N = 9843$ is very large, so we could not see from the histogram and sample skewness $\hat{S}(X) = 0.061387$ that the distribution has some small positive skewness.

3. Testing for Heavy tails. Next we test if the excess kurtosis $K(X) - 3$ of the distribution of returns is significantly different from 0.

Note: the sample kurtosis $\hat{K}(X) = 12.91$ is much larger than 3. The zero excess kurtosis $K(X) - 3 = 0$ means that distribution does not have heavy tails, whereas $K(X) > 3$ would indicate that distribution has heavy tails (we call it leptokurtic), and secondly, it is not a normal distribution. (For normal distribution always $K(X) = 3$).

We have $\hat{K}(X) = 12.91$ and the number of observations $N = 9843$.

We test the hypothesis at 5% significance level:

H_0: $K(X) - 3 = 0$ against alternative H_1: $K(X) \neq 3$.

We use the test statistics:

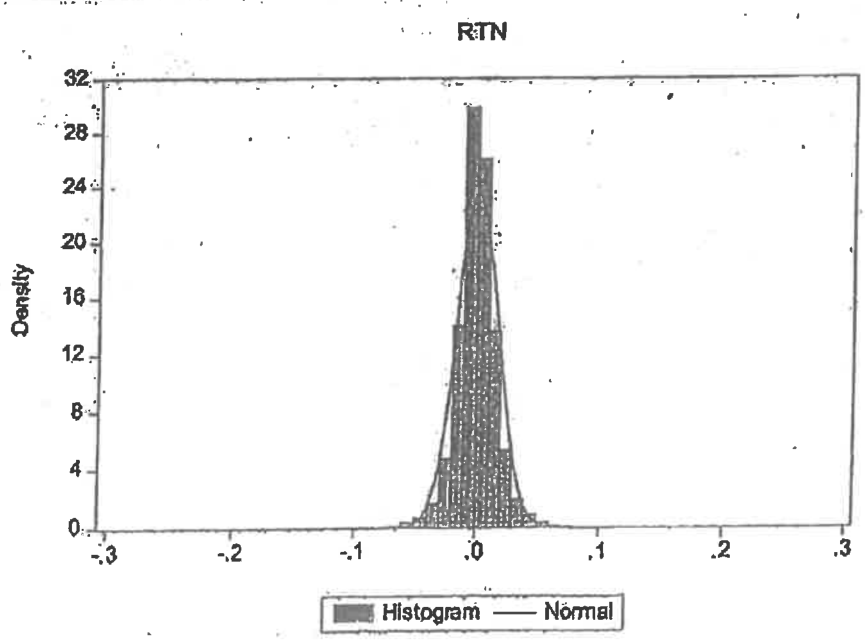
$$t = \frac{\hat{K}(X) - 3}{\sqrt{24/N}} = \frac{12.91 - 3}{\sqrt{24/9843}} = 200.6928.$$

By theory, under null hypothesis, $t \sim N(0,1)$ is normally distributed. Therefore the testing rule is similar as for testing skewness:

Rule: reject H_0 at 5% significance level, if

$$|t| \geq z_{2.5\%} \sim 2.$$

In our case $t = 200.6928 > 2$. Hence, the test rejects the null hypothesis of zero kurtosis $K(X) = 3$.
Comment: we found that $K(X) > 3$, the distribution has heavy tails, and it is not normal.



4. Jargue-Bèra test (for normal distribution). In the e-views output you find that the Jargue-Bera statistic takes value 40335.5 and the corresponding p-value is 0.0000 (in the output it is called "probability")

Jargue-Bera test is a test for normality of distribution. It combines testing for skewness and access kurtoses into one test.

Jargue-Bera test is used to test the hypothesis:

H_0: $S(X) = 0$ and $K(X) - 3 = 0$ ("normal distribution")

against alternative

H_1: $S(X) \neq 0$ or $K(X) - 3 \neq 0$ ("distribution is not normal").

Normal distribution has $S(X) = 0$ and $K(X) = 3$. Thus, in case of normal distribution, test will not reject H_0.

Test will reject H_0 if either skewness is not 0 or if kurtosis is not 3. That will indicate that distribution is not normal.

Since the $p = 0.0000$ value of Jargue-Bera test is smaller than 0.05 we reject the asymptotic normality at significance level 5%.

(c) Histogram and the normal probability distribution with the mean = sample mean and the SD= "sample SD", when plotted together, show that the normal density does not fit well to the histogram in the peak region. In case of the normal distribution, we would expect histogram and the normal density to be closer each other.

From the histogram we see that the tails of the histogram are heavier than the tails of the normal density. This can be explained by the fact that the sample kurtosis $\hat{K}(X) = 12.9$ is much higher than the kurtosis of a normal distribution which equals to 3.