Topic : GRAPHICAL ANALYSIS

PURPOSES OF STATISTICAL GRAPHS

- DISCOVERY
- ANALYSIS
- DETECTIVE WORK

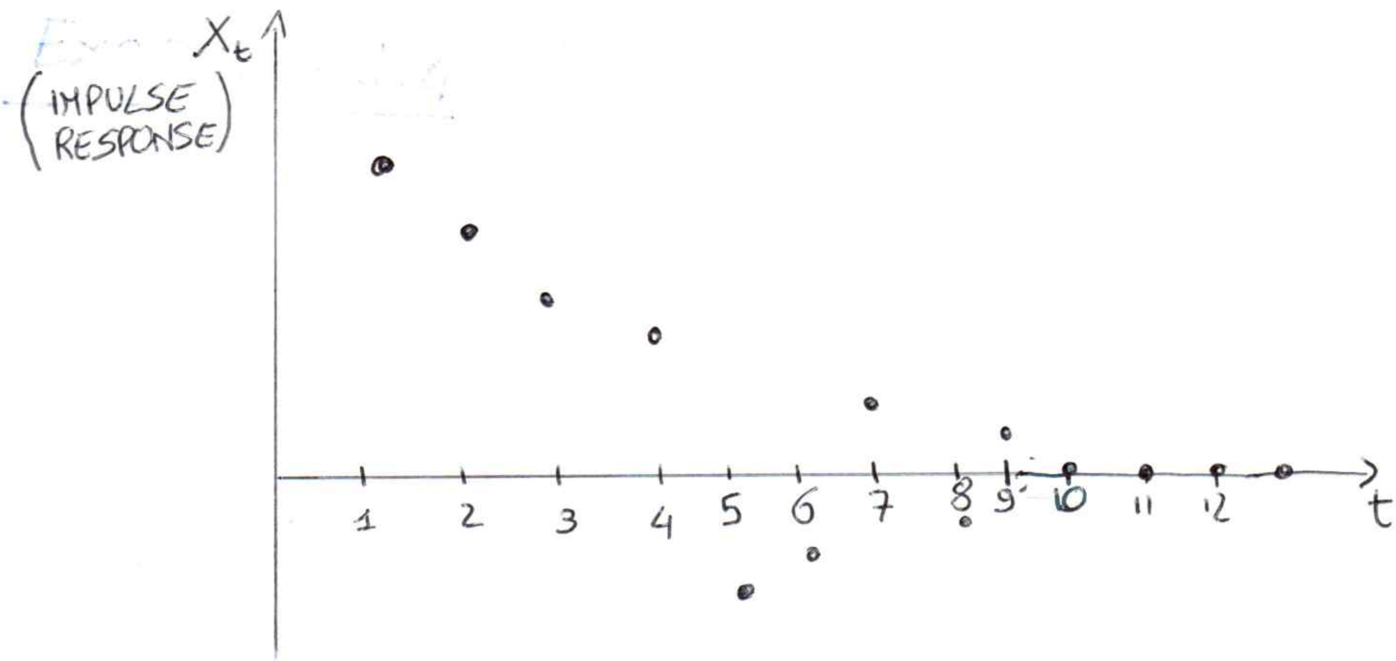
COMMUNICATION OF PERTINENT FINDINGS AND PATTERNS IN THE DATA

- COMMUNICATES KEY POINTS
- USE MINIMUM AMOUNT OF NOTES IN THE DATA REGION
- COMPACT GRAPHS ARE OFTEN USEFUL
- GRAPH CONSTRUCTION IS AN ITERATIVE PROCESS
- SCALE USED IN THE GRAPH IS CRUCIAL

Topic: IMPULSE RESPONSE FUNCTION

A STATIONARY TIME SERIES X_t CAN BE SEEN AS THE IMPULSE RESPONSE FUNCTION TO RANDOM SHOCKS ε_t.

IT PROVIDES US WITH INFORMATION HOW THE IMPULSE PROPAGATES THROUGH THE SYSTEM AND WHAT EFFECT IT HAS OVER TIME.



Exercise 1.4

COMPUTE THE IMPULSE RESPONSE FUNCTION

$$X_t = 0.6\, X_{t-1} - 0.2\, X_{t-2} + \varepsilon_t$$

for $t = 1, 2, 3, 4, 5$

WHEN WE HAVE A SINGLE SHOCK $\varepsilon_0 = 1$ AT TIME 0, WHEREAS $\varepsilon_1 = \varepsilon_2 = \ldots = 0$.

ASSUME ALSO THAT $X_{-1} = 0$ AND $X_{-2} = 0$

• Step 1 : COMPUTE X_0, X_1, X_2, X_3, X_4 AND X_5

• $X_0 = 0.6\, \underbrace{X_{-1}} - 0.2\, \underbrace{X_{-2}} + \underbrace{\varepsilon_0}$

we know that:

$X_{-1} = 0$

$X_{-2} = 0$

$\varepsilon_1 = 1$

$= 0.6(0) - 0.2(0) + 1 = \underline{\underline{1}}$

• $X_1 = 0.6\, X_0 - 0.2\, X_{-1} + \varepsilon_1$

$= 0.6(1) - 0.2(0) + 0 = \underline{\underline{0.6}}$

in the step above we obtained $X_0 = 1$

$\varepsilon_1 = \varepsilon_2 = \ldots = 0$

$\cdot\ X_2 = 0.6\,X_1 - 0.2\,X_0 + \varepsilon_2$

$$= 0.6\,(0.6) - 0.2\,(1) + 0 = \underline{\underline{0.16}}$$

$x_1 = 0.6$ $\qquad$ $x_0 = 1$

$\cdot\ X_3 = 0.6\,X_2 - 0.2\,X_1 + \varepsilon_3$

$$= 0.6\,(0.16) - 0.2\,(0.6) + 0 = \underline{\underline{-0.0240}}$$

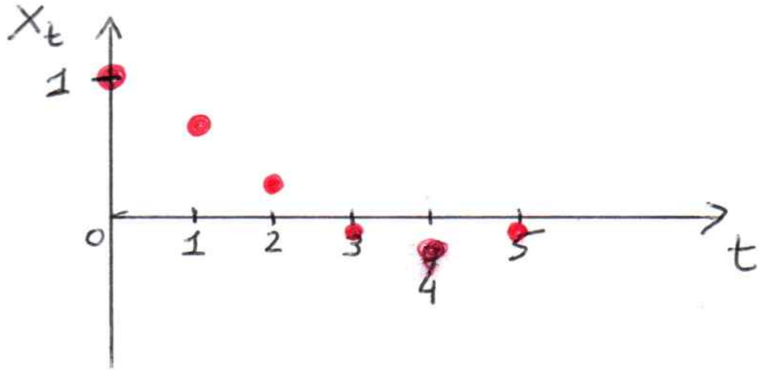
$\cdot\ X_4 = 0.6\,X_3 - 0.2\,X_2 + \varepsilon_4$

$$= 0.6\,(-0.0240) - 0.2\,(0.16) + 0 = \underline{\underline{-0.0464}}$$

$\cdot\ X_5 = 0.6\,X_4 - 0.2\,X_3 + \varepsilon_5$

$$= 0.6\,(-0.0464) - 0.2\,(-0.0240) + 0 = \underline{\underline{-0.0230}}$$

step 2 : GRAPH



- IMPULSE RESPONSE FUNCTION TENDS TO ZERO AS t INCREASES.
- IMPULSE RESPONSE FUNCTION TAKES POSITIVE AND NEGATIVE VALUES.

step 3 : HOW MANY LAGS ARE NEEDED FOR THE IMPACT OF SHOCK TO BE REDUCED BY 50 % ?

- THE DECREASE OF THE IMPACT OF THE SHOCK AT TIME 0 AFTER 1 LAG:

$$\frac{X_1 - X_0}{X} \cdot 100\% = \frac{0.6 - 1}{1} \cdot 100\% = -40\%$$

- THE DECREASE OF THE IMPACT OF THE SHOCK AT TIME 0 AFTER 2 LAGS:

$$\frac{X_2 - X_0}{X_0} \cdot 100\% = \frac{0.16 - 1}{1} \cdot 100\% = -84\%$$

SO THE IMPACT OF SHOCK WILL BE REDUCED BY 50% AFTER TWO LAGS.

Exercise 1.5

COMPUTE THE IMPULSE RESPONSE FUNCTION

$X_t = \phi X_{t-1} + \varepsilon_t$ for $t = 0, 1, 2, 3, 4, 5$

WE HAVE $\varepsilon_0 = 1$ AND $\varepsilon_1 = \varepsilon_2 = \dots = 0$

WE ASSUME $X_{-1} = 0$

a) LET $\phi = 0.7$

$X_t = \phi X_{t-1} + \varepsilon_t$

for $t = 0$ $\quad X_0 = 0.7 X_{-1} + \varepsilon_0$

$= 0.7(0) + 1 = 1$

for $t = 1$ $\quad X_1 = 0.7 X_0 + \varepsilon_1$

$= 0.7(1) + 0 = 0.7$

for $t = 2$ $\quad X_2 = 0.7 X_1 + \varepsilon_2 =$

$= 0.7(0.7) + 0 = 0.49$

for $t = 3$ $\quad X_3 = 0.7 X_2 + \varepsilon_t$

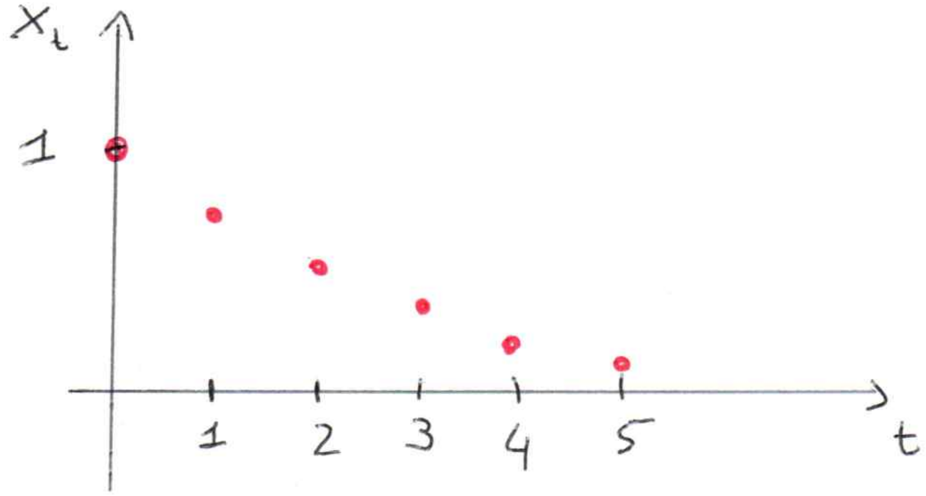
$= 0.7(0.49) + 0 = 0.3430$

for $t=4$ $\quad X_4 = 0.7 X_3 + \varepsilon_4$

$$= 0.7\,(0.3430) + 0 = 0.2401$$

for $t=5$ $\quad X_5 = 0.7\, X_4 + \varepsilon_5$

$$= 0.7\,(0.2401) + 0 = 0.1681$$



THE IMPULSE RESPONSE X_t TENDS TO ZERO FAST, MONOTONICALLY, AND TAKES NON-NEGATIVE VALUES.

b) LET $\phi = 1$

$$X_t = 1X_{t-1} + \varepsilon_t$$

for $t=0$ $\quad X_0 = 1X_{-1} + \varepsilon_0 = 1(0) + 1 = 1$

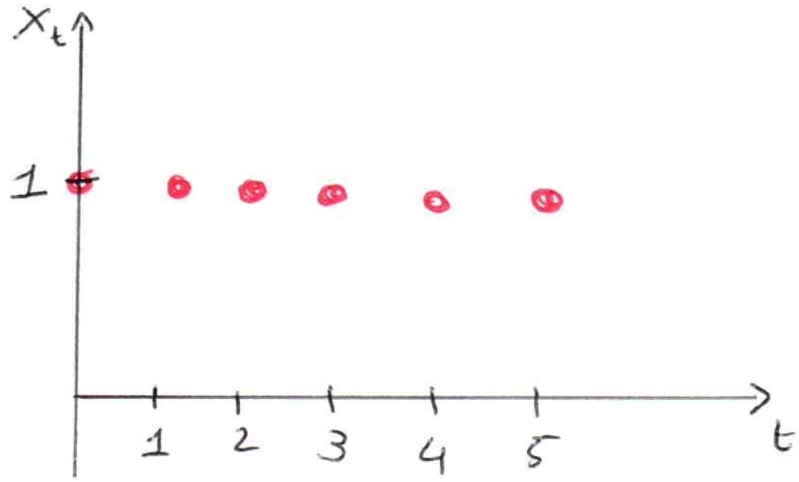
for $t=1$ $\quad X_1 = 1X_0 + \varepsilon_1 = 1(1) + 0 = 1$

for $t=2$ $\quad X_2 = 1X_1 + \varepsilon_2 = 1(1) + 0 = 1$

for $t=3$ $\quad X_3 = 1X_2 + \varepsilon_3 = 1(1) + 0 = 1$

for $t=4$ $\quad X_4 = 1X_3 + \varepsilon_4 = 1(1) + 0 = 1$

for $t=5$ $\quad X_5 = 1X_4 + \varepsilon_5 = 1(1) + 0 = 1$



THE SHOCK DOES NOT DECREASE OVER TIME

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THE SHOCK STAYS IN THE SYSTEM FOREVER