Solutions of ECOM073 TEST 2024

Question 1 (33 marks)

(i) Test for absence of correlation using the following Eview output.

Use all available information given in this output.

Justify and explain your answers.

The sample size is $N = 400$.

Correlogram of R

Date: 06/11/20 Time: 12:56
Sample: 1 400
Included observations: 400

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.032	0.032	0.4132	0.520
		2	0.071	0.070	2.4559	0.293
		3	0.034	0.030	2.9181	0.404
		4	0.072	0.065	5.0071	0.287
		5	-0.006	-0.014	5.0211	0.413
		6	0.056	0.046	6.2865	0.392
		7	0.007	0.001	6.3071	0.504
		8	-0.021	-0.032	6.4796	0.594
		9	0.033	0.032	6.9152	0.646
		10	0.013	0.008	6.9898	0.726

Solution of question 1.

(i) (**Testing for correlation using ACF**.) Time series is a white noise if it is serially uncorrelated, that is $\rho_k = 0$ for $k \geq 1$. Hence, to test for white noise, we test the hypotheses

$H_0 : \rho_k = 0$ against alternative $H_1 : \rho_k \neq 0$

at each lag $k = 1, 2, ...$ at significance level 5%.

Rule: ACF ρ_k at lag k is significantly different from zero at 5% significance level if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then ACF at lag k is not significantly different from 0.

(ii) **Ljung-Box test**. This test can be also used to test for zero correlation. We select $m = 1, 2, ...$ and test the hypothesis

$H_0 : \rho_1 = ...\rho_m = 0$ against alternative

$H_1 : \rho_j \neq 0$ for some $j = 1, ..., m$.

We reject the H_0 at 5% significance level, if $p-$ value satisfies $p < 0.05$. If time series is white noise, that we do not reject H_0 for any $m = 1, 2, ...$

(iii) We have $2/\sqrt{N} = 2/\sqrt{400} = 0.1$. Since $|\rho_1| = 0.023 < 0.1$, $|\rho_2| = 0.032 < 0.1$, ..., and so on. We find that ρ_k is not significant at any lag $k = 1, 2, ...$ at 5% significance level, because $|\rho_k| \leq 2/\sqrt{N} = 2/\sqrt{400} = 0.1$. So, this time series is a white noise.

From the correlogram we see that p-values of Ljung Box test satisfy $p > 0.05$ for all $m = 1, ...$ So H_0 is not rejected at any m and this time series is a white noise.

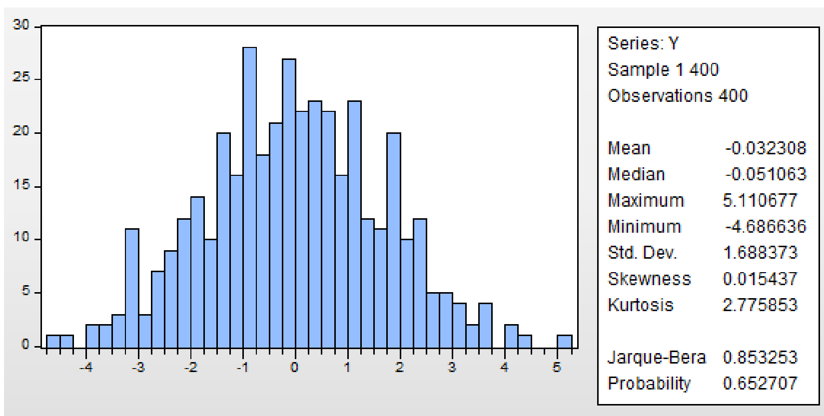
Question 2 (34 marks)

(i) Explain what is meant by a "stationary time series".

(ii) Test for significance of the skewness and excess kurtosis of returns r_t using summary statistics given below. Explain your testing procedures.

What can you say about the distribution of r_t?

The sample size is $N = 400$.



Solution of Question 2.

(i) Time series X_t is a covariance stationary time series if it satisfies three properties:

– $EX_t = \mu$ for all t (does not depend on t);

– $Var(X_t) = \sigma^2$ for all t (does not depend on t);

– $Cov(X_t, X_{t-k}) = \gamma_k$ - covariance function depends only on the lag k and does not depend on t.

In case of a non-stationary time series, one of these properties will be not satisfied, for example the mean EX_t could vary in time.

(ii) **Testing whether skewness** $S(X) = 0$ **(symmetry of distribution)**.

We test the hypothesis

H_0: $S(X) = 0$ against alternative H_1: $S(X) \neq 0$.

at 5% significance level.

We construct the test statistics:

$$t = \frac{\hat{S}(X)}{\sqrt{6/N}} = \frac{0.01543}{\sqrt{6/400}} = 0.12599$$

Under the null hypothesis, $t \sim N(0, 1)$ is normally distributed.

Rule: we reject H_0 at 5% significance level, if

$$|t| \geq 2.$$

In our case $|t| = 0.12599 < 2$. Hence, the test shows that there no evidence in the data to reject the null hypothesis of zero skewness $S(X) = 0$.

Testing whether kurtosis $K(x) = 3$ **(non heavy tails)**.

Next we test at 5% significance level the hypothesis:

H_0: $K(X) - 3 = 0$ against alternative H_1: $K(X) \neq 3$.

We use the test statistics:

$$t = \frac{\hat{K}(X) - 3}{\sqrt{24/N}} = \frac{2.775 - 3}{\sqrt{24/400}} = -0.91856.$$

By theory, under null hypothesis, $t \sim N(0, 1)$ is normally distributed. Therefore the testing rule is similar as for testing skewness:

<u>Rule</u>: reject H_0 at 5% significance level, if

$$|t| \geq 2.$$

In our case $|t| = 0.91856 < 2$. Hence, the test does not rejects the null hypothesis of zero kurtosis $K(X) = 3$.

Jargue-Bera test for asymptotic normality. Jargue-Bera test is used to test the hypothesis:

H_0: $S(X) = 0$ and $K(X) - 3 = 0$ ("normal distribution")

against alternative

H_1: $S(X) \neq 0$ or $K(X) - 3 \neq 0$ ("distribution is not normal").

Normal distribution has $S(X) = 0$ and $K(X) = 3$. Thus, in case of normal distribution, test will not reject H_0.

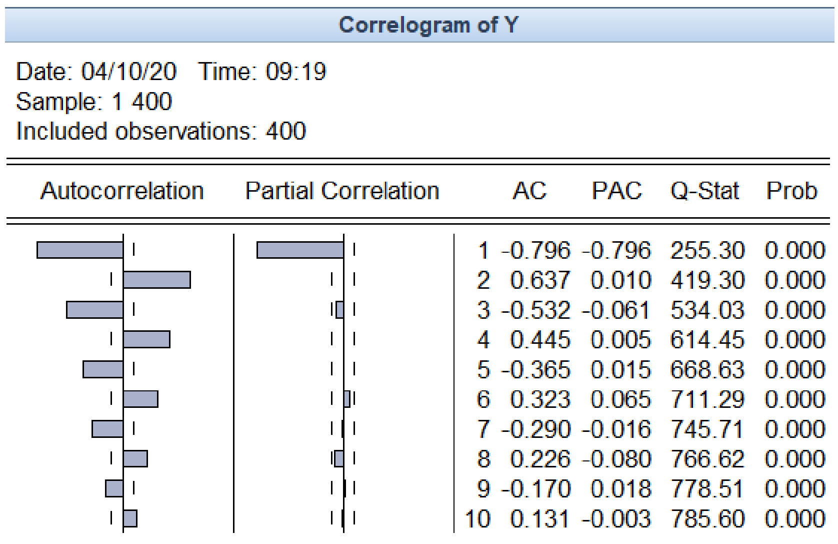
Test will reject H_0 if either skewness is not 0 or if kurtosis is not 3. That will indicate that distribution is not normal.

Since the $p = 0.6527$ value of Jargue-Bera test is larger than 0.05 we do not reject the asymptotic normality at significance level 5%.

Question 3 (33 marks)

(i) Using the given Eviews output determine the order of AR(p) and MA(q) model you would fit to the data. Explain your answer.

The sample size is $N = 400$.



Correlogram of Y

Date: 04/10/20 Time: 09:19
Sample: 1 400
Included observations: 400

	AC	PAC	Q-Stat	Prob
1	-0.796	-0.796	255.30	0.000
2	0.637	0.010	419.30	0.000
3	-0.532	-0.061	534.03	0.000
4	0.445	0.005	614.45	0.000
5	-0.365	0.015	668.63	0.000
6	0.323	0.065	711.29	0.000
7	-0.290	-0.016	745.71	0.000
8	0.226	-0.080	766.62	0.000
9	-0.170	0.018	778.51	0.000
10	0.131	-0.003	785.60	0.000

Solution of Question 3:

(i) To select the order p for AR(p) model, we use the sample PACF function. We test the hypothesis

$H_0 : \rho_k = 0$ against alternative $H_1 : \rho_k \neq 0$

at lags $k = 1, 2, ...$ at significance level 5%, where ρ_k is the PACF function.

PACF $\hat{\rho}_k$ at lag k is significantly different from 0 at 5% significance level, if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then PACF at lag k is not significantly different from 0.

Rule: we select for p the largest lag k at which the PACF is significant.

This rule can be used because PACF of the AR(p) model becomes 0 for $k > p$.

—- To select the order q of MA(q) model, we use the sample ACF function. We test the hypothesis

$H_0 : \rho_k = 0$ against alternative $H_1 : \rho_k \neq 0$

at lags $k = 1, 2, ...$ at significance level 5%, where ρ_k is ACF function.

Rule: ACF ρ_k is significant at lag k at 5% significance level, if $|\hat{\rho}_k| > 2/\sqrt{N}$, where N is the number of observations.

If $|\hat{\rho}_k| \leq 2/\sqrt{N}$, then ACF at lag k is not significantly different from 0.

We select for q the largest lag k at which the ACF is significant.

(ii) We have $2/\sqrt{N} = 2/\sqrt{400} = 0.1$. The PACF is significant only at lag 1. Hence we would fit AR(1) model.

The ACF shows significant correlation at the lags 1 to 10. Hence we would fit MA(10) model.

From the two models AR(1) and MA(10) we select a simpler model AR(1) with smaller number of parameters which should to be fitted to the data.

(iii) According to (ii), we can fit AR(1) model

$$X_t = \phi_0 + \phi X_{t-1} + \varepsilon_t$$

where ε_t is white noise.

(iv) This AR(1) model would fit the data if residuals $\hat{\varepsilon}_t = X_t - \widehat{\phi} X_{t-1}$ are uncorrelated. We could use the correlogram of residuals to test whether residuals are uncorrelated.

Quiz 5: mini problem 1

Question 1.

Suppose $X_1, ..., X_t$ is a sample from a stationary MA(1) time series

$$X_t = 0.2X_{t-1} + \varepsilon_t,$$

where ε_t is an i.i.d. sequence with zero mean and variance 1.

- **(a)** Find the 1-step ahead forecast $\hat{X}_t(1)$ of X_{t+1}, the forecast error and the variance of the forecast error.
- **(b)** Find the 2-step ahead forecast $\hat{X}_t(2)$ of X_{t+2}, the forecast error and the variance of the forecast error.
- **(c)** What can you say about the k-step ahead forecast $\hat{X}_t(1)$ this time series?

Solution. Consider AR(1) time series model

$$X_t = \phi X_{t-1} + \varepsilon_t$$

where $|\phi| < 1$.

(a) We use the following forecasting rules. The k-step ahead forecast is defined by the formula

$$\hat{X}_t(k) = E[X_{t+k}|F_t].$$

We will uses the rules:

$E[\varepsilon_{t+1}|F_t] = 0$, $E[\varepsilon_{t+2}|F_t] = 0$, $E[\varepsilon_{t+3}|F_t] = 0$,...

$E[X_t|F_t] = X_t$, $E[X_{t-1}|F_t] = X_{t-1}$, ... since by time t, we already know X_t, X_{t-1}, ...

$E[\varepsilon_t|F_t] = \varepsilon_t$, $E[\varepsilon_{t-1}|F_t] = \varepsilon_{t-1}$, ...

Recall that we use notation

$$[X_{t+k}] = E[X_{t+k}|F_t]$$

To compute $\hat{X}_t(1)$ first we write

$$X_{t+1} = \phi X_t + \varepsilon_t.$$

Then, using forecasting rules we obtain

$$\hat{X}_t(1) = E[X_{t+1}|F_t] = [X_{t+1}] = [\phi X_t + \varepsilon_{t+1}] = \phi[X_t] + [\varepsilon_{t+1}] = \phi X_t$$

since $[X_t] = E[X_t|F_t] = X_t$ and $[\varepsilon_{t+1}] = E[\varepsilon_{t+1}|F_t] = 0$.

Therefore, the 1-step ahead forecast is $\hat{X}_t(1) = \phi X_t$.

For $\phi = 0.2$, we obtain, $\hat{X}_t(1) = 0.2X_t$.

The error of 1-step ahead forecast is

$$e_t(1) \quad = \quad X_{t+1} - \hat{X}_t(1) = (\phi X_t + \varepsilon_{t+1}) - \phi X_t = \varepsilon_{t+1}.$$

The variance of the error is $Var(e_t(1)) = Var(\varepsilon_{t+1}) = \sigma_\varepsilon^2 = 1$.

(b) To compute 2-step ahead forecast, write

$$X_{t+2} = \phi X_{t+1} + \varepsilon_{t+2}.$$

Then

$$\begin{aligned} \hat{X}_t(2) &= E[X_{t+2}|F_t] = [X_{t+2}] = [\phi X_{t+1} + \varepsilon_{t+2}] = \phi[X_{t+1}] + [\varepsilon_{t+2}] \\ &= \phi X_t(1) + 0 = \phi(\phi X_t) = \phi^2 X_t. \end{aligned}$$

The forecast error:

$$e_t(2) = X_{t+2} - \hat{X}_t(2) = (\phi X_{t+1} + \varepsilon_{t+2}) - \phi X_t(1) = \phi(X_{t+1} - X_t(1)) + \varepsilon_{t+2} = \phi\varepsilon_{t+1} + \varepsilon_{t+2}.$$

The variance of the forecast error

$$Var(e_t(2)) = E(\phi\varepsilon_{t+1} + \varepsilon_{t+2})^2 = E[\phi^2\varepsilon_{t+1}^2] + E[\varepsilon_{t+2}^2] = \phi^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2.$$

We see that

$$Var(e_t(2)) = \phi^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2 \geq Var(e_t(1)) = \sigma_\varepsilon^2$$

that is the variance of the 2-step ahead forecast error is larger than the variance of the 1-step ahead forecast error.

(c) Because of the mean reversion property, as k increases,

$$X_t(k) \to EX_t.$$

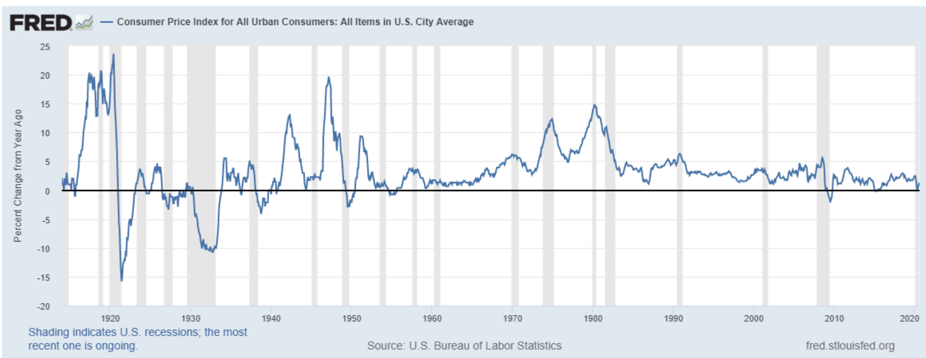
The average value of our time series is $EX_t = 0$. Therefore,

$$X_t(k) \to 0.$$

Quiz 6:   mini problems 1,2,3

Question 1.

Using given plot, comment whether this time series is stationary or non-stationary.



Solution. This time series does not seem to have a constant mean, so it is a non-stationary time series.

Question 2.

Suppose $Y_1, ..., Y_t$ is a sample from a time series

$$Y_t = Y_{t-1} + x_t, \quad x_t = \phi x_{t-1} + \varepsilon_t$$

where ε_t is an i.i.d. sequence with zero mean and variance 1 and $|\phi| < 1$.

- **(a)** Find the 1-step ahead forecast $\hat{Y}_t(1)$ of Y_{t+1}, the forecast error and the variance of the forecast error.
- **(b)** Find the 2-step ahead forecast $\hat{Y}_t(2)$ of Y_{t+2}, the forecast error and the variance of the forecast error.
- **(c)** Suggest a forecast for Y_{t+20}.

Solution. (a) We have

$$Y_{t+1} = Y_t + X_{t+1}.$$

Note that

$$E[X_{t+1}|F_t] = E[\phi X_t + \varepsilon_{t+1}|F_t] = \phi X_t.$$

Thus

$$\begin{aligned} \hat{Y}_t(1) &= E[Y_{t+1}|F_t] \\ &= E[Y_t + X_{t+1}|F_t] \\ &= E[Y_t|F_t] + E[X_{t+1}|F_t] \\ &= Y_t + \phi X_t. \end{aligned}$$

The 1-step ahead forecast errors is

$$\begin{aligned} e_t(1) &= Y_{t+1} - \hat{Y}_t(1) \\ &= Y_t + X_{t+1} - (Y_t + \phi X_t) \\ &= \varepsilon_{t+1}. \end{aligned}$$

The variance of the 1-step ahead forecast errors is

$$Var(e_t(1)) = Var(\varepsilon_{t+1}) = \sigma_\varepsilon^2.$$

(b) We have

$$Y_{t+2} = Y_{t+1} + x_{t+2}.$$

Note that

$$E[x_{t+2}|F_t] = E[\phi x_{t+1} + \varepsilon_{t+2}|F_t] = E[\phi x_{t+1}|F_t] + E[\varepsilon_{t+2}|F_t] = \phi E[x_{t+1}|F_t] = \phi^2 x_t.$$

Thus

$$\begin{aligned} \hat{Y}_t(2) &= E[Y_{t+2}|F_t] \\ &= E[Y_{t+1} + x_{t+2}|F_t] \\ &= E[Y_{t+1}|F_t] + E[x_{t+2}|F_t] = \hat{Y}_t(1) + \hat{x}_t(2) \\ &= Y_t + \phi x_t + \phi^2 x_t = Y_t + x_t(\phi + \phi^2) = Y_t + (Y_t - Y_{t-1})(\phi + \phi^2) \end{aligned}$$

since $x_t = Y_t - Y_{t-1}$.

The 2-step ahead forecast errors is

$$\begin{aligned} e_t(2) &= Y_{t+2} - \hat{Y}_t(2) \\ &= Y_{t+1} + x_{t+2} - (\hat{Y}_t(1) + \hat{x}_t(2)) \\ &= (Y_{t+1} - \hat{Y}_t(1)) + (x_{t+2} - \hat{x}_t(2)). \end{aligned}$$

We showed that

$$e_t(1) = Y_{t+1} - \hat{Y}_t(1) = \varepsilon_{t+1}.$$

We have

$$\begin{aligned} x_{t+2} - \hat{x}_t(2) &= X_{t+2} - \phi^2 x_t = \phi x_{t+1} + \varepsilon_{t+2} - \phi^2 x_t = \phi(\phi x_t + \varepsilon_{t+1}) + \varepsilon_{t+2} - \phi^2 x_t \\ &= \phi\varepsilon_{t+1} + \varepsilon_{t+2}. \end{aligned}$$

Therefore,

$$e_t(2) = \varepsilon_{t+1} + \phi\varepsilon_{t+1} + \varepsilon_{t+2} = \varepsilon_{t+1}(1+\phi) + \varepsilon_{t+2}.$$

The variance of the 2-step ahead forecast errors is

$$Var(e_t(2)) = Var(\varepsilon_{t+1}(1+\phi)+\varepsilon_{t+2}) = Var(\varepsilon_{t+1}(1+\phi))+Var(\varepsilon_{t+2}) = (1+\phi)^2\sigma_\varepsilon^2+\sigma_\varepsilon^2.$$

(c) Since Y_t is a unit root time series, forecasting Y_{t+20}, 20- step ahead would produce a large forecast error. So, not good forecast can be suggested.

Question 3.

Consider time series

$$Y_t = \mu + Y_{t-1} + \varepsilon_{t-1},$$

where ε_t is a white noise sequence with zero mean and variance 1. Suppose that $Y_0 = 1$.

- Find $E[Y_t]$
- $Var(Y_t)$.

Solution.

(a) We can write write

$$\begin{aligned} Y_t &= \mu + Y_{t-1} + \varepsilon_t = \mu + (\mu + Y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\ &= 2\mu + Y_{t-2} + \varepsilon_{t-1} + \varepsilon_t \\ &= 3\mu + Y_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t \\ &= .... \\ &= t\mu + Y_0 + \varepsilon_t + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_1 \\ &= t\mu + 1 + \varepsilon_t + \varepsilon_{t-2} + \varepsilon_{t-1} + ... + \varepsilon_1, \end{aligned}$$

since $Y_0 = 1$. Then

$$\begin{aligned} E[Y_t] &= E[\mu t + 1 + \varepsilon_t + \varepsilon_{t-2} + ... + \varepsilon_1] \\ &= \mu t + 1 + E[\varepsilon_t] + E[\varepsilon_{t-2}] + ... + E[\varepsilon_1] \\ &= \mu t + 1 + 0 + 0 + ... + 0 = \mu t + 1. \end{aligned}$$

(b)

$$\begin{aligned} Var(Y_t) &= E[(Y_t - E[Y_t])^2] = E[(t\mu + 1 + \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + ... + \varepsilon_1 - \mu t - 1)^2] \\ &= E[(\varepsilon_t + \varepsilon_{t-1} + ... + \varepsilon_1)^2] \\ &= E[\varepsilon_t^2] + E[\varepsilon_{t-1}^2] + ... + E[\varepsilon_1^2] = \sigma_\varepsilon^2 + \sigma_\varepsilon^2 + ... + \sigma_\varepsilon^2 = t\sigma_\varepsilon^2 \end{aligned}$$

noting that ε_t is a white noise and therefore

$E[\varepsilon_i \varepsilon_j] = 0$ if $i \neq j$;

$E[\varepsilon_i \varepsilon_j] = \sigma_\varepsilon^2$ if $i = j$.

(c) By definition,

$$Cov(Y_t, Y_s) = E[(Y_t - E[Y_t])(Y_s - E[Y_s])].$$

Let $t \geq s$.
Since $E[Y_t] = \mu t + 1$ and

$$Y_t - E[Y_t] = \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + ... + \varepsilon_1,$$

we obtain

$$\begin{aligned} Cov(Y_t, Y_s) &= E[(\varepsilon_t + \varepsilon_{t-1} + ... + \varepsilon_1)(\varepsilon_s + \varepsilon_{s-1} + ... + \varepsilon_1)] \\ &= E[(\{\varepsilon_s + \varepsilon_{t-1} + ... + \varepsilon_1\} + \{\varepsilon_{s+1} + ... + \varepsilon_t\})(\varepsilon_s + \varepsilon_{s-1} + ... + \varepsilon_1)] \\ &= E[(\varepsilon_s + \varepsilon_{t-1} + ... + \varepsilon_1)^2] + E[(\varepsilon_{s+1} + ... + \varepsilon_t)(\varepsilon_s + \varepsilon_{s-1} + ... + \varepsilon_1)] \\ &= s\sigma_\varepsilon^2 + 0 = s\sigma_\varepsilon^2. \end{aligned}$$

Therefore

$$Cov(Y_t, Y_s) = s\sigma_\varepsilon^2 = \min(t, s)\sigma_\varepsilon^2.$$