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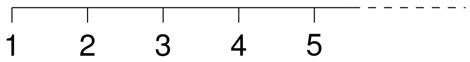

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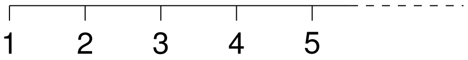

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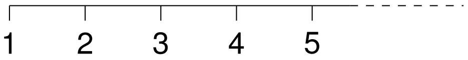

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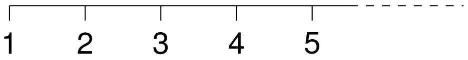

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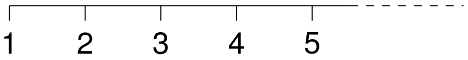

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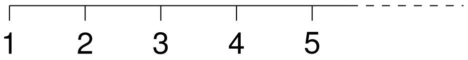

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We introduce the integers to get round this.

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- The ordering $<$ can be extended to $\mathbb{Z}$, and still satisfies familiar rules.

Divisibility

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Lemma 4.1: Suppose $a, b, c \in \mathbb{N}$. If $a \mid b$ and $b \mid c$ then $a \mid c$.