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e.g. let $P(x)$ denote the statement "$x^2 \leqslant 2$".
Then $P(1)$ is true, but $P(2)$ is false.

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e.g. if P = "Paris is in France" and Q = "London is in Brazil", then

"P and Q" says "Paris is in France and London is in Brazil", which is false
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(The table should have one row for each combination of true/false for each of the statements P, Q.)

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(P and Q) or R is not equivalent to P and (Q or R). Brackets are important!

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The last two examples show that “and” and “or” get swapped by negation: not (P and Q) is equivalent to (not P) or (not Q).

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P	Q	$P \Rightarrow Q$
true	true	true
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Variations in wording

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"If P then Q" can be written as

- Q if P
- P implies Q
- Q is implied by P
- P only if Q
- P is sufficient for Q
- Q is necessary for P
- $P \Rightarrow Q$.

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- $(n \geqslant 4) \Rightarrow (n \geqslant 3)$ is a true implication
- (n is prime) $\Rightarrow$ (n is odd) is a false implication.