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If $z = a + b\mathrm{i}$, then $a$ is the real part of $z$, and $b$ is the imaginary part.

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$$(-\mathrm{i})(-\mathrm{i}) = -1.$$

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So to divide by $c + di$, we just multiply by $\frac{c-di}{c^2+d^2}$.

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To divide by $c + di$, we just multiply by $\dfrac{c - di}{c^2 + d^2}$.

$$\frac{3 + 2i}{3 + 4i} = \frac{(3 + 2i)(3 - 4i)}{3^2 + 4^2} = \frac{17 - 6i}{25} = \frac{17}{25} - \frac{6}{25}i$$

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$$\frac{4+2\mathrm{i}}{2-\mathrm{i}} = \frac{(4+2\mathrm{i})(2+\mathrm{i})}{2^2+1^2} = \frac{8}{5}\mathrm{i}$$