Reminder: Learning support hour
11:30–12:30 on Fridays
In my office: maths 512.
Please come along if you have any questions about the module.

Manipulating sums

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$$\sum_{n=1}^{10}(n^2-6) = \sum_{n=1}^{9}(n^2-6) \; + \; 10^2-6$$

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So

$$\sum_{n=0}^{6}(n+1)^3 = \sum_{m=1}^{7} m^3$$

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Now let $m = n + 1$ in the first sum, and $m = n$ in the second:

$$S = \sum_{m=2}^{100} m^3 \; - \; \sum_{m=1}^{99} m^3.$$

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Now spit off the terms $m = 100$ and $m = 1$:

$$\begin{aligned} S &= \sum_{m=2}^{99} m^3 + 100^3 - \sum_{m=2}^{99} m^3 - 1^3 \\ &= 999999. \end{aligned}$$

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The techniques we learned for manipulating sums can all be used with products too.

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(We also define $0! = 1$.)

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- ✗ “$2 + 4$.”
- ✗ “The set of all buses in London.”