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We can put eventually before any of these properties to mean that the sequence has that property after a certain point.

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  This is weakly decreasing and eventually constant.

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