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is the sequence $((-2)^k)_{k \in \mathbb{N}}$.

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1, 4, 9, 16, 25, 36, ...

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The sequence $(b_k)_{k=1}^{\infty}$, where $b_k = 4k^2$.