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Part (c) is often used in counting problems: if we want to find $|A|$ for some set A, we find a bijection $A \to B$ for a set B for which we know $|B|$.

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