# Range

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

$$\mathrm{range}(f) = \{f(a) \ : \ a \in A\} .$$

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

$$\mathrm{range}(f) = \{f(a) \ : \ a \in A\}\,.$$

**Examples:**

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

$$\text{range}(f) = \{f(a) \ : \ a \in A\} .$$

**Examples:**

- $g : \{1, 2, 3, 4\} \to \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$ defined by

$$g(1) = \heartsuit, \;\; g(2) = \heartsuit, \;\; g(3) = \clubsuit, \;\; g(4) = \diamondsuit.$$

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

$$\text{range}(f) = \{f(a) \ : \ a \in A\}\,.$$

**Examples:**

- $g : \{1, 2, 3, 4\} \to \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$ defined by

$$g(1) = \heartsuit, \quad g(2) = \heartsuit, \quad g(3) = \clubsuit, \quad g(4) = \diamondsuit.$$

$$\text{range}(g) = \{\heartsuit, \clubsuit, \diamondsuit\}.$$

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

$$\text{range}(f) = \{f(a) \ : \ a \in A\} .$$

**Examples:**

- $g : \{1, 2, 3, 4\} \to \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$ defined by

$$g(1) = \heartsuit, \quad g(2) = \heartsuit, \quad g(3) = \clubsuit, \quad g(4) = \diamondsuit.$$

  $\text{range}(g) = \{\heartsuit, \clubsuit, \diamondsuit\}$.

- $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = \sin(x)$.

# Range

Suppose $f : A \to B$. The range of $f$ is the set of all values that $f$ takes.

$$\mathrm{range}(f) = \{f(a) \ : \ a \in A\}.$$

**Examples:**

- $g : \{1, 2, 3, 4\} \to \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\}$ defined by

$$g(1) = \heartsuit, \quad g(2) = \heartsuit, \quad g(3) = \clubsuit, \quad g(4) = \diamondsuit.$$

  $\mathrm{range}(g) = \{\heartsuit, \clubsuit, \diamondsuit\}$.

- $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = \sin(x)$.
  $\mathrm{range}(h) = [-1, 1]$.

# Injective, surjective, bijective

# Injective, surjective, bijective

Suppose $f : A \to B$.

# Injective, surjective, bijective

Suppose $f : A \to B$. Then $f$ is:

# Injective, surjective, bijective

Suppose $f : A \to B$. Then $f$ is:

- injective if different elements of the domain are mapped to different elements of the codomain

# Injective, surjective, bijective

Suppose $f : A \to B$. Then $f$ is:

- injective if different elements of the domain are mapped to different elements of the codomain
  i.e. if $a_1, a_2 \in A$ and $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$

# Injective, surjective, bijective

Suppose $f : A \to B$. Then $f$ is:

- injective if different elements of the domain are mapped to different elements of the codomain
  i.e. if $a_1, a_2 \in A$ and $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$
- surjective if the range equals the codomain

# Injective, surjective, bijective

Suppose $f : A \to B$. Then $f$ is:

- injective if different elements of the domain are mapped to different elements of the codomain
  i.e. if $a_1, a_2 \in A$ and $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$
- surjective if the range equals the codomain
  i.e. for every $b \in B$, there is $a \in A$ such that $f(a) = b$

# Injective, surjective, bijective

Suppose $f : A \to B$. Then $f$ is:

- injective if different elements of the domain are mapped to different elements of the codomain
  i.e. if $a_1, a_2 \in A$ and $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$
- surjective if the range equals the codomain
  i.e. for every $b \in B$, there is $a \in A$ such that $f(a) = b$
- bijective if it is both injective and surjective.

# Proof tips

Suppose $f : A \to B$.

# Proof tips

Suppose $f : A \to B$.

- To prove $f$ is not surjective, give an explicit $b$ such that there is no $a$ for which $f(a) = b$.

# Proof tips

Suppose $f : A \to B$.

- To prove $f$ is not surjective, give an explicit $b$ such that there is no $a$ for which $f(a) = b$.
- To prove $f$ is not injective, give explicit $a_1, a_2$ such that $a_1 \neq a_2$ but $f(a_1) = f(a_2)$.

# Proof tips

Suppose $f : A \to B$.

- To prove $f$ is not surjective, give an explicit $b$ such that there is no $a$ for which $f(a) = b$.
- To prove $f$ is not injective, give explicit $a_1, a_2$ such that $a_1 \neq a_2$ but $f(a_1) = f(a_2)$.
- To prove $f$ is surjective, show how to find, for general $b$, an element $a$ such that $f(a) = b$.

# Proof tips

Suppose $f : A \to B$.

- To prove $f$ is not surjective, give an explicit $b$ such that there is no $a$ for which $f(a) = b$.
- To prove $f$ is not injective, give explicit $a_1, a_2$ such that $a_1 \neq a_2$ but $f(a_1) = f(a_2)$.
- To prove $f$ is surjective, show how to find, for general $b$, an element $a$ such that $f(a) = b$.
- To prove $f$ is injective, prove the contrapositive: show that if $f(a_1) = f(a_2)$, then $a_1 = a_2$.

# Examples

# Examples

- $f : \mathbb{N} \to \mathbb{N}$ defined by $x \mapsto x^2$.

# Examples

- $f : \mathbb{N} \to \mathbb{N}$ defined by $x \mapsto x^2$.
  $f$ is injective but not surjective.

# Examples

- $f : \mathbb{N} \to \mathbb{N}$ defined by $x \mapsto x^2$.
  $f$ is injective but not surjective.

- $f : \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\} \to \{\text{red}, \text{blue}, \text{green}\}$ defined by

$$\spadesuit \longmapsto \text{blue}, \qquad \heartsuit \longmapsto \text{red}, \qquad \diamondsuit \longmapsto \text{blue}, \qquad \clubsuit \longmapsto \text{green}.$$

# Examples

- $f : \mathbb{N} \to \mathbb{N}$ defined by $x \mapsto x^2$.
  $f$ is injective but not surjective.

- $f : \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\} \to \{\text{red}, \text{blue}, \text{green}\}$ defined by

$$\spadesuit \longmapsto \text{blue}, \qquad \heartsuit \longmapsto \text{red}, \qquad \diamondsuit \longmapsto \text{blue}, \qquad \clubsuit \longmapsto \text{green}.$$

  $f$ is surjective but not injective.

# Examples

- $f : \mathbb{N} \to \mathbb{N}$ defined by $x \mapsto x^2$.
  $f$ is injective but not surjective.

- $f : \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\} \to \{\text{red}, \text{blue}, \text{green}\}$ defined by

  $$\spadesuit \longmapsto \text{blue}, \qquad \heartsuit \longmapsto \text{red}, \qquad \diamondsuit \longmapsto \text{blue}, \qquad \clubsuit \longmapsto \text{green}.$$

  $f$ is surjective but not injective.

- $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(x)$.

# Examples

- $f : \mathbb{N} \to \mathbb{N}$ defined by $x \mapsto x^2$.
  $f$ is injective but not surjective.

- $f : \{\spadesuit, \heartsuit, \diamondsuit, \clubsuit\} \to \{\text{red}, \text{blue}, \text{green}\}$ defined by

  $$\spadesuit \longmapsto \text{blue}, \qquad \heartsuit \longmapsto \text{red}, \qquad \diamondsuit \longmapsto \text{blue}, \qquad \clubsuit \longmapsto \text{green}.$$

  $f$ is surjective but not injective.

- $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(x)$.
  $f$ is neither injective nor surjective.