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More ways to write a set

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$\{1, 2, \ldots, n\}$ is the set of all integers from 1 to n.

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"The set of all rational numbers less than 4".

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$\emptyset$ is the empty set: the set with no elements.

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$\left\{x \in \mathbb{Q} \ : \ x^2 < 0\right\}$ is the empty set.

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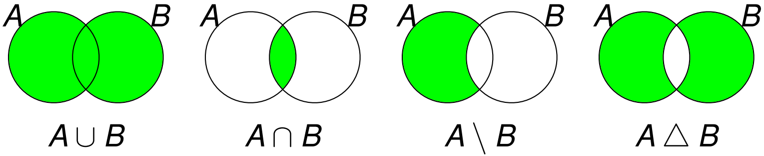

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Standard way to prove that two sets S, T are equal: prove that $S \subseteq T$ and $T \subseteq S$.