Complex Networks (MTH6142)
Solutions of Formative Assignment 10

- **The average number of friends of someone friends**
  Given a random uncorrelated network with degree distribution $P(k)$.
  a) Show that

$$\frac{\left\langle k^2 \right\rangle}{\langle k \rangle} \geq \langle k \rangle \tag{1}$$

  where the equal sign holds only for the regular networks (all the nodes with the same degree).
  b) Motivate that this result justify the social network paradox summarized in the following sentence: *Your friends have more friends than you do!!* Assume for simplicity that the social network is not regular, and that the network is uncorrelated.

- *Notes on solution*
  a) For any uncorrelated random network we have

$$\begin{aligned} \left\langle (k - \langle k \rangle)^2 \right\rangle &\geq 0 \\ \left\langle k^2 \right\rangle - \langle k \rangle^2 &\geq 0 \\ \frac{\langle k^2 \rangle}{\langle k \rangle} &\geq \langle k \rangle. \end{aligned} \tag{2}$$

  where the equality holds if and only if

$$P(k) = \begin{cases} 1 & \text{if} \quad k = z \\ 0 & \text{if} \quad k \neq z \end{cases} \tag{3}$$

  or in other words only if the network is regular, i.e.

$$k_i = z = \langle k \rangle \tag{4}$$

  for every node i of the network $i = 1, 2, \ldots, N$. In fact only in this case we have

$$\langle k^2 \rangle = z^2 = \langle k \rangle^2. \tag{5}$$

  b) The average degree of the neighbour of a node is given in an uncorrelated random network by

$$\sum_k k q(k) = \sum_k k \frac{k}{\langle k \rangle} P(k) = \frac{\langle k^2 \rangle}{\langle k \rangle} \tag{6}$$

The average degree of random nodes in the network is given by $\langle k \rangle$.

For any uncorrelated random network that is not a regular network we have

$$\frac{\langle k^2 \rangle}{\langle k \rangle} > \langle k \rangle. \tag{7}$$

Therefore the average number of friends of a neighbour of a node is larger than the average degree of the network!