



Complex Networks (MTH6142) Formative Assignment 9

- **1. Diameter of Cayley trees**

A Cayley tree is a symmetric regular tree constructed starting from an origin node of degree k . In a Cayley tree every node at distance d from the origin node has degree k , until we reach the nodes at distance \mathcal{P} , which have degree equal to 1 and are called the leaves of the network.

In this question take a Cayley tree with $k = 3$ as shown in the figure, and consider an arbitrary value of \mathcal{P} .

- a) Show using induction that the number of nodes at distance d from the central node is $3 \times 2^{d-1}$ for $d \in [1, \mathcal{P}]$.
- b) Using the formula for the sum of the first terms of a geometric series, show that the total number of nodes in the network is given by

$$N = 1 + 3 [2^{\mathcal{P}} - 1].$$

- c) Show that the diameter D is given by $D = 2\mathcal{P}$.
- d) Hence find an expression for the diameter D of the network in terms of the total number of nodes N .
- e) Consider the expression obtained in point (d) and find the leading term of D in terms of the total number of nodes N in the network when $N \gg 1$.
- f) Does the network display the small-world distance property?
- g) Which is the clustering coefficient of this network?

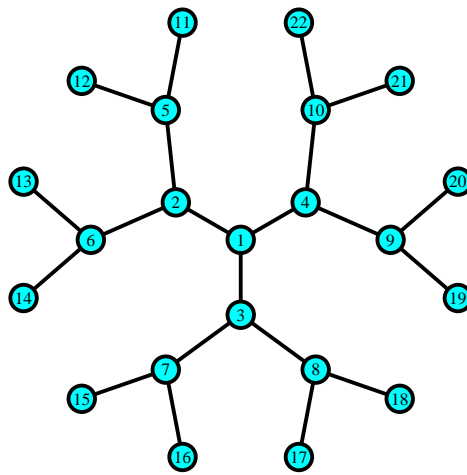


Figure 1: A Cayley tree with $k = 3$ and $\mathcal{P} = 3$.