WEEK 11 Lecture 2

SUMMING UP: Cayley trees and also random (Poisson) networks

✓ $\boxed{A}$ have the S.W.D.P.

~~$\boxed{B}$~~ DO NOT HAVE high clustering coefficient

Instead REAL-WORLD NETWORKS

	N	$\langle k \rangle$	ℓ	ℓ^{RAND}	C	C^{RAND}
C.elegans neural network	282	14	2.65	2.25	0.28	0.05
Power-grids	4941	2.67	18.7	12.4	0.08	0.005
Internet (snapshots)	3015-6209	3.52-4.11	3.7-3.75	6.36-6.18	0.18-0.3	0.001
WWW (snapshot)	153127	35.21	3.1	3.35	0.1078	0.00023
World, synonyms	22311	13.48	4.5	3.84	0.7	0.006

A + B

are SMALL-WORLD NETWORKS

We can construct sparse networks with high C $\longrightarrow$ Property $\boxed{B}$

7.4 REGULAR 1D LATTICES

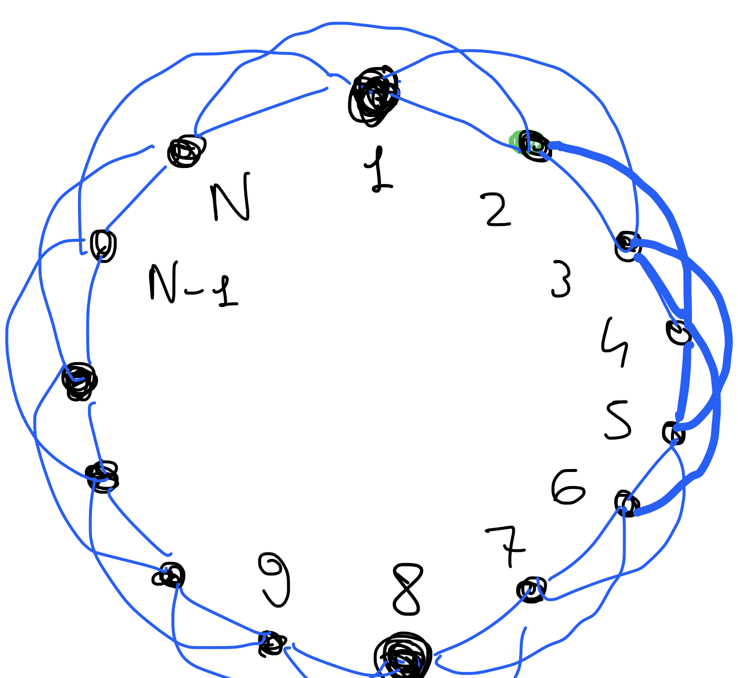
DEF

N even   K even

A 1D regular lattice with N nodes of degree K is a ring of N nodes $i \in \{1, 2, \dots, N\}$ such that i and j are linked if they are at distance $\leq \frac{K}{2}$ on the ring

$$|i-j| \leq \frac{K}{2}$$

$N = 14 \qquad K = 4$

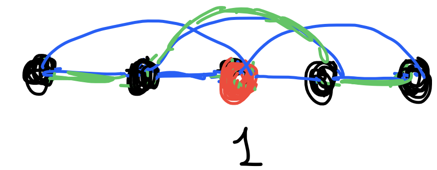


C_i is the same for each i

Let us calculate C_1

$$C_1 = \frac{T_1}{\frac{k_1(k_1-1)}{2}} = \frac{\cancel{3}}{\frac{4 \cdot \cancel{3}}{2}} = \frac{1}{2}$$

2



$k_1 = 4$

$T_1 = 3$

PROPOSITION

A 1D REGULAR LATTICE with N nodes of degree k has:

$$C_{WS} = \frac{3}{4}\,\frac{k-2}{k-1}$$

No proof

Check $k=4 \to C = \frac{3}{4}\,\frac{4-2}{4-1} = \frac{\cancel{3}}{4}\,\frac{2}{\cancel{3}} = \frac{1}{2}$ ✓

NOTICE that $C^{REG\ LATT}$ is independent of N

Therefore for $N \gg 1$ $\quad C^{Reg\ Latt} = \frac{3}{4}\,\frac{k-2}{k-1} \gg C^{Rand} = \frac{\langle k \rangle}{N}$

Hence 1D regular lattices have a high clustering coefficient

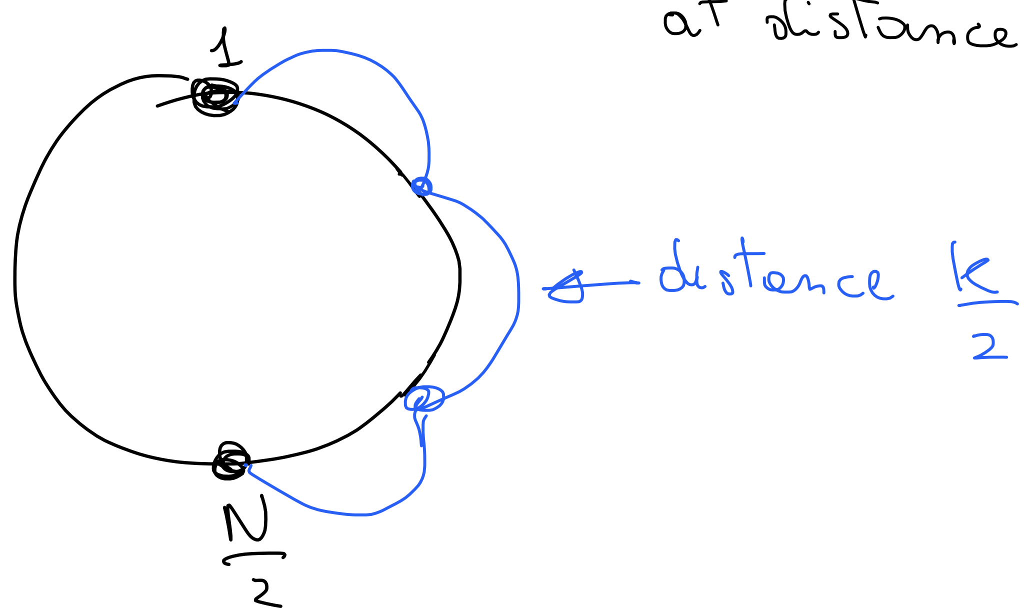
PROPOSITION

A 1D REGULAR LATTICE with N nodes and degree K has

$$\boxed{D \simeq \frac{N}{K} \qquad \ell \simeq \frac{N}{2K} \qquad \text{for } N \gg 1}$$

Proof

The pair of nodes further apart in the network are the ones at distance $\frac{N}{2}$ over the ring



Since nodes at distance $\frac{K}{2}$ on the ring are linked

$$D \simeq \frac{\frac{N}{\cancel{2}}}{\frac{K}{\cancel{2}}} = \frac{N}{K}$$

SUMMING UP

1 D regular lattices

~~A~~ DO NOT HAVE the SWDP

✓ B have high clustering coeff

7.7 SMALL-WORLD NETWORK MODELS

DEF

(N, K even)

Starting from a 1D regular lattice with N nodes of degree K, each link is removed with probability p $(0 \le p \le 1)$ and is randomly reconnected. For intermediate values of p you have networks with high clustering coeff and S.W.D.P.

B A

P=0 REG LATT

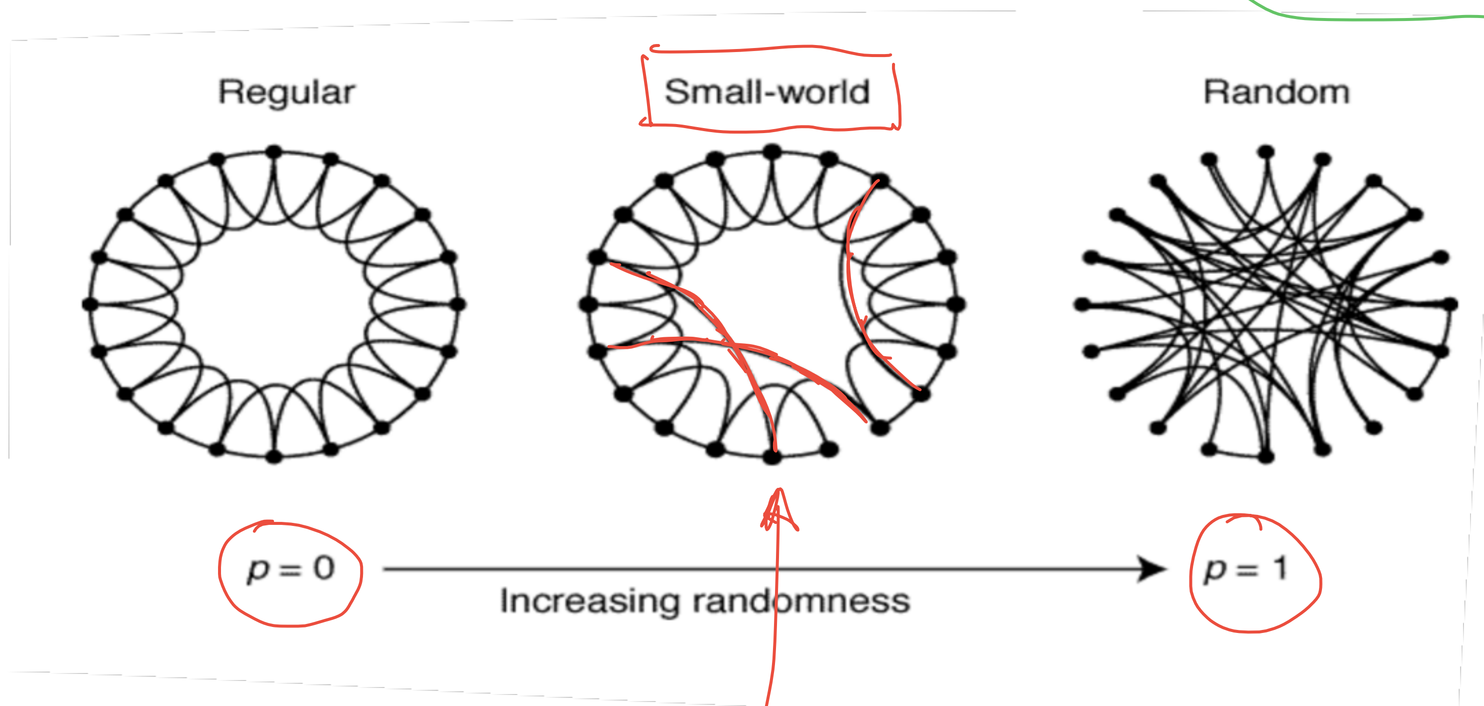
C is finite

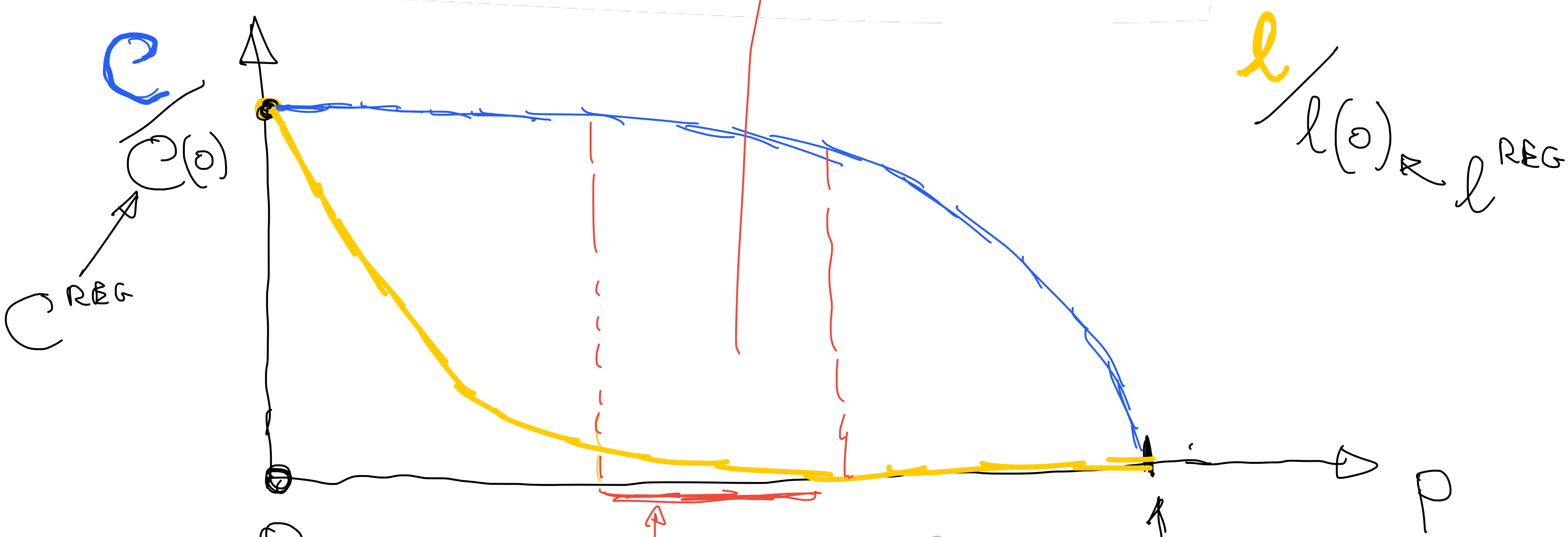
$L \sim N$

P=1 RAND NET

$C \sim \frac{1}{N}$

$L \sim \ln N$





SMALL-WORLD NETWORKS