Queen Mary
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Complex Networks (MTH6142)
Formative Assignment 8

- **1. Growing network model with uniform attachment**
  At time $t = 0$ the network is formed by two nodes joined by a link.
  - At every time step a single new node joins the network, so that at time t there will be exactly $N(t) = 2 + t$ nodes.
    Every new node has initially $m = 1$ links.
  - Each new link attaches to an existing node of the network. The target node i is chosen with probability Π_i following a uniform attachment rule $\Pi_i = \frac{1}{N(t)}$, where k_i is the degree of the node i.

a) What is the time evolution of the degree $k_i(t)$ of the generic node i in the mean-field approximation?

b) What is the degree distribution of the network at large times in the mean-field approximation?

c) What is the master equation for the average number of nodes with degree k?

d) Solve the master equation finding the exact degree distribution of the network.

- **2. The Bianconi-Barabási model with uniform fitness distribution**
  Each node i of the network has a fitness value η_i drawn from a uniform distribution $\rho(\eta) = 1$ for $\eta \in [0, 1]$.
  At time $t = 0$ the network is formed by two nodes joined by a link.
  - At every time step a single new node j with fitness η_j drawn from the $\rho(\eta)$ distribution joins the network, so that at time t there will be exactly $N = 2 + t$ nodes.Every new node has initially $m = 1$ links.
  - Each new link attaches to an existing node of the network. The target node i is chosen with probability Π_i following the preferential attachment rule biased toward nodes of high degree and high fitness $\Pi_i = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$, where k_i is the degree of the node i.

  a) Assume that $\sum_j \eta_j k_j \simeq Ct$ for $t \gg 1$, with $C > 0$ independent on the time t. What is the time evolution of the degree $k_i(t)$ of the generic node i in the mean-field approximation?

  b) Check self-consistently that the assumption made in point a) is correct and determine the equation that the constant C needs to satisfy.

  c) What is the degree distribution of the network at large times in the mean-field approximation?