



Complex Networks (MTH6142) Formative Assignment 6

The moments of a power-law degree distribution

Consider a network of N nodes with power-law degree distribution

$$P(k) = Ck^{-\gamma} \quad (1)$$

where C is the normalization constant and the power-law exponent γ is greater than one, i.e. $\gamma > 1$.

Assume that the maximum degree K is given by

$$K = \min(N, N^{1/(\gamma-1)}) \quad (2)$$

and the minimum degree is given by $k_{min} = 1$.

- a) Evaluate, in the continuous approximation, the value of the normalization constant C determined by the equation

$$1 = \int_1^K dk P(k). \quad (3)$$

- b) Evaluate, in the continuous approximation, $\langle k \rangle$ and $\langle k^2 \rangle$.
- c) Perform the limit $N \rightarrow \infty$ of $\langle k \rangle$ and $\langle k^2 \rangle$ obtained in part b) for $\gamma \in (1, 2]$.
- d) Perform the limit $N \rightarrow \infty$ of $\langle k \rangle$ and $\langle k^2 \rangle$ obtained in part b) for $\gamma \in (2, 3]$.
- e) Perform the limit $N \rightarrow \infty$ of $\langle k \rangle$ and $\langle k^2 \rangle$ obtained in part b) for $\gamma \in (3, \infty)$.
- f) When is the network scale-free?