WEEK 8 Tutorial

FEEDBACK ON QUIZ 3

Q: RANDOM GRAPHS

Consider $G(N,p)$ with $p = \frac{e^{\ln 2}}{3N}$

- In the limit $N \to \infty$, the average degree is given by

∞, 0, $\frac{2}{3}$, NONE of THESE

$$\langle k \rangle = p(N-1) = \frac{e^{\ln 2}}{3N}(N-1) \xrightarrow[N\to\infty]{} \frac{e^{\ln 2}}{3} = \frac{2}{3}$$

→ Therefore the graph

HAS NOT, HAS a giant component when $N \to \infty$

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SUPERCRITICAL (i.e. there is a giant component)

$$\text{if } \lim_{N\to\infty} \langle k \rangle > 1$$

in our case $\lim_{N\to\infty} \langle k \rangle = \frac{2}{3} < 1 \Rightarrow$ NO GIANT

Q: DEGREE DISTRIBUTION

- A random graph $G(N,p)$ with $N = 1000$ and $p = \frac{1}{2}$ has a $P(k)$ given by

POISS, BIN, POWER-LAW, UNIFORM, ALL of the ABOVE

$G(N,p)$ has always a Binomial degree distribution

$$P_B(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

with $\langle k \rangle = p(N-1)$

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A random graph $G(N,p)$ with $N = 10^{11}$ $p = 2 \cdot 10^{-11}$

Poiss, Exp, Unif, All of the Above

Again the answer is BINOMIAL DISTRIBUTION

but since N is large and $p = 2 \cdot 10^{-11} = \frac{2}{10^{11}} = \frac{2}{N}$

POISSON NETWORKS $\rightarrow$ $p = \frac{c}{N-1}$

Hence the degree distribution of the graph in the question is well approximated by a

Poisson distribution $P_p(k) = \frac{c^k e^{-c}}{k!} = \frac{2^k e^{-2}}{k!}$

Q: BINOMIAL

Which random graph $G(N,p)$ has $\langle k\rangle = 100$ and variance σ_k^2 of the degree distribution given by $\sigma_k^2 = 90$

$p=0.1$, $N=9001$	$p=\frac{1}{9}$, $N=901$	$\boxed{p=\frac{1}{10},\ N=1001}$	NONE of THESE

$$P_B(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

See tutorial W5 for formulas

$$\langle k\rangle = p(N-1)$$

$$\sigma_k^2 = (N-1)p(1-p)$$

$$\sigma_k^2 = \langle k\rangle (1-p)$$

$$90 = 100(1-p)$$

$$\frac{90}{100} = 1-p \qquad p = 1-\frac{90}{100} = 0.1$$

$p=0.1$

still 2 solutions one possible

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But we also want $\langle k \rangle = p(N-1)$

So $N = \frac{\langle k \rangle}{p} + 1 = \frac{100}{0.1} + 1 = 1000 + 1 = 1001$

$N = 1001$

Q: POISSON

You want to use a random graph $G(N, p)$ with $N = 10^{13}$ to best approximate a Poisson random graph with $\langle k \rangle = 100$. Which p would you use?

$\boxed{p = 10^{-11}}$ $\quad p = 0.001 \quad p = 0.000001 \quad p = 10^{-9}$

Poisson with $c = \langle k \rangle = 100$

$\langle k \rangle = p(N-1) = 100$

$pN \simeq 100 \qquad p = \frac{100}{N} = \frac{10^2}{10^{13}} = 10^{-11}$

Q: DATA

Consider a data set of $N = 10^6$ nodes and $L = 6 \times 10^5$ links. Assuming that the data is well described by a random graph model would you expect to find a GIANT COMPONENT in the network?

YES NO

The condition to have a giant component in random graph models is

$$\lim_{N \to \infty} \langle k \rangle > 1$$

In our case $\langle k \rangle = \frac{2L}{N} = \frac{2 \cdot 6 \cdot 10^5}{10^6} = \frac{12}{10} = 1.2 > 1$

Yes we expect a giant component

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FA 6

- **The moments of a power-law degree distribution**
  Consider a network of N nodes with power-law degree distribution

$$P(k) = Ck^{-\gamma} \tag{1}$$

where C is the normalization constant and the power-law exponent γ is greater than one, i.e. $\gamma > 1$.
Assume that the maximum degree K is given by

$$K = \min(N, N^{1/(\gamma-1)}) \tag{2}$$

and the minimum degree is given by $k_{min} = 1$.

a) Evaluate, in the continuous approximation, the value of the normalization constant C determined by the equation

$$1 = \int_1^K dk P(k). \tag{3}$$

b) Evaluate, in the continuous approximation, $\langle k \rangle$ and $\langle k^2 \rangle$.

c) Perform the limit $N \to \infty$ of $\langle k \rangle$ and $\langle k^2 \rangle$ obtained in part b) for $\gamma \in (1, 2]$.

d) Perform the limit $N \to \infty$ of $\langle k \rangle$ and $\langle k^2 \rangle$ obtained in part b) for $\gamma \in (2, 3]$.

e) Perform the limit $N \to \infty$ of $\langle k \rangle$ and $\langle k^2 \rangle$ obtained in part b) for $\gamma \in (3, \infty)$.

f) When is the network scale-free?

Important Info : Assessed coursework 4

opens today Friday, 15th March

→ you will need to submit a handwritten document uploading it as a PDF document

→ manually marked