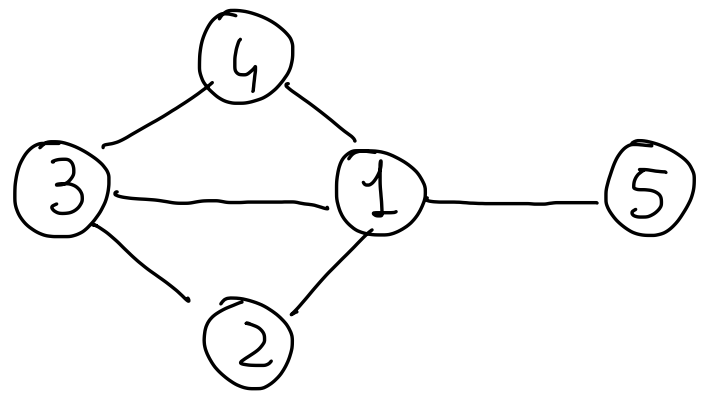
WEEK 5 tutorial

FEEDBACK on QUIZ 2

Q: CENTRALITY



$N = 5$

(a) Which is the diameter of this network?

1, 2, 0, NONE

$$D = \max_{i,j} \{ d_{ij} \} \longrightarrow D = 2$$

you can calculate matrix $dl = \{ d_{ij} \}$

distances between i and j or from i to j

1

$dl =$

	1	2	3	4	5
1	0	1	1	1	1
2	1	0	1	2	2
3	1	1	0	1	2
4	1	2	1	0	2
5	1	2	2	2	0

D

(b) Which is the efficiency of node 3?

$\frac{5}{6}$, $\frac{5}{2}$, $\frac{7}{8}$, NONE

$$E_i = \frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{1}{d_{ij}}$$

$$E_3 = \frac{1}{4}\left[\frac{1}{d_{31}} + \frac{1}{d_{32}} + \frac{1}{d_{34}} + \frac{1}{d_{35}}\right] = \frac{1}{4}\left[1 + 1 + 1 + \frac{1}{2}\right] =$$

$$= \frac{1}{4} \cdot \frac{7}{2} = \frac{7}{8}$$

(c) Which is the betweenness of node 3?

7, 5, 8, 10

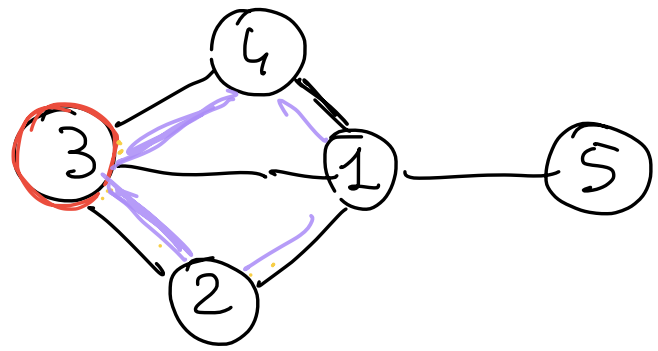
(2)

$$b_i = \sum_{r,s} \frac{n^i_{rs}}{g_{rs}}$$

n^i_{rs} ← # of shortest paths between r and s PASSING by node i

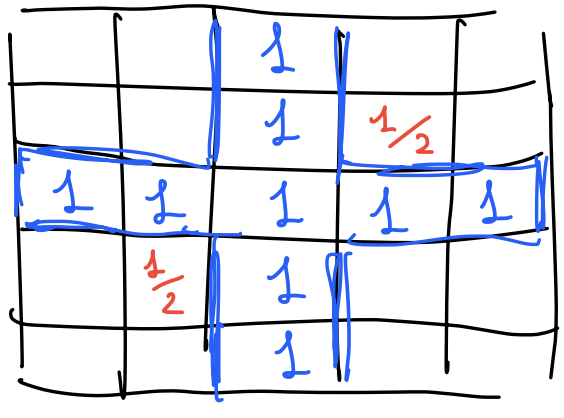
g_{rs} ← # of shortest paths between r and s

$$b_3 = \sum_{r,s} \frac{n^3_{rs}}{g_{rs}}$$



	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	2	1
3	1	1	1	1	1
4	1	2	1	1	1
5	1	1	1	1	1

	1	2	3	4	5
1	0	0	1	0	0
2	0	0	1	1	0
3	1	1	1	1	1
4	0	1	1	0	0
5	0	0	1	0	0



CROSS

$2N - 1 = 9$

$$b_3 = 9 + \frac{1}{2} + \frac{1}{2} = 10$$

- Calculate b_5 — 5 is a leaf → $2N-1 = 9$
- Calculate b_4 — you have to repeat the calculation

Q: EULER-HAMILTON

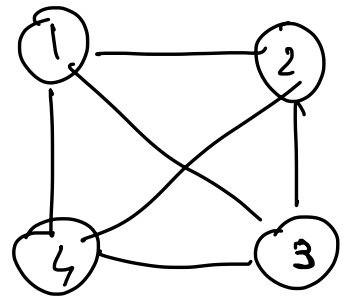
DEF EULERIAN CYCLE

A cyclic path that traverses each LINK of the network exactly once

DEF HAMILTONIAN CYCLE

A cyclic path that traverses each NODE of the network exactly once

(a) Consider a complete network with 4 nodes



Does this network have an Eulerian cycle?

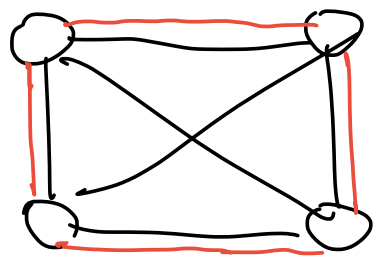
Y (N)

EULER THEO

Eulerian cycle $\Leftrightarrow$ k_i is EVEN $\forall i$

$k_i = 3 \quad i = 1, 2, 3, 4$ all nodes have ODD degree $\Rightarrow$ NO

(b) Does this network have an Hamiltonian cycle? (Y) N



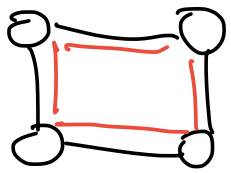
yes

(c) Is it possible that an Hamiltonian cycle is also an Eulerian cycle of this network?

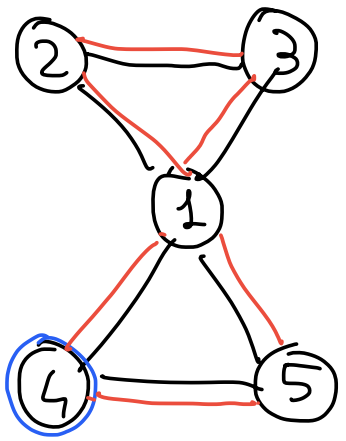
Y N (N/A)

This network does not have Eulerian cycles

Other examples in other networks

G_1 

This cycle is
- Eulerian
- Hamiltonian

G_2 

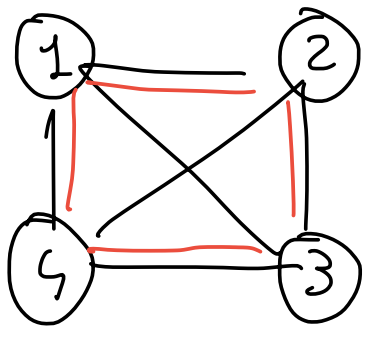
4, 5, 1, 2, 3, 1, 4

This is
- Eulerian
- Not Hamiltonian

G_3

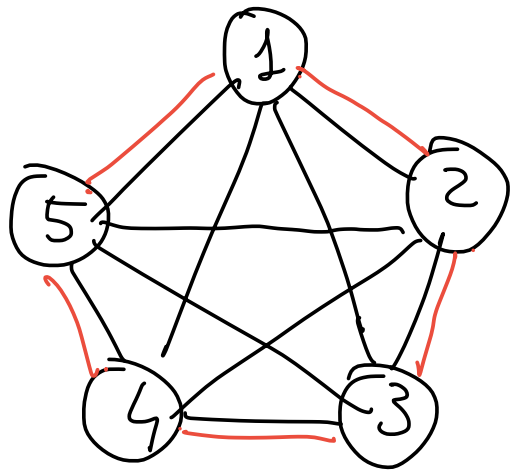
$N = 4$

Complete



This is
- Not Eulerian
- Hamiltonian

G_4



$N=5$

Complete

The cycle in red is → NOT Eulerian
→ Hamiltonian

Notice: G_4 is an Eulerian graph

since $k_i = 4 \quad \forall i$ ← EVEN

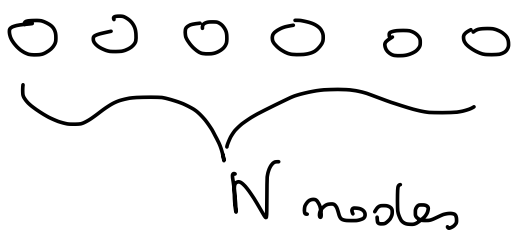
However it is not possible that in G_4

an Hamiltonian cycle is also an Eulerian cycle

Q: MIN/MAX # of LINKS

(a) What is the min # of links L of an undirected network of N nodes?

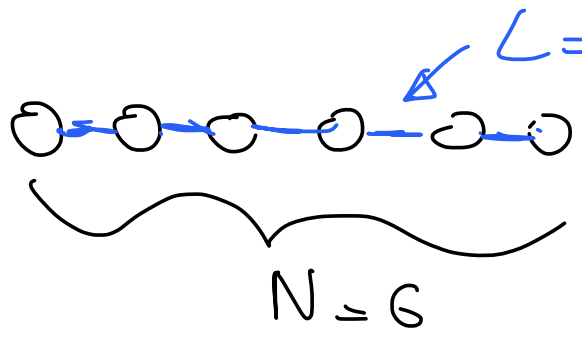
(○) $N+1$ N NONE



No links $\Rightarrow L = 0$

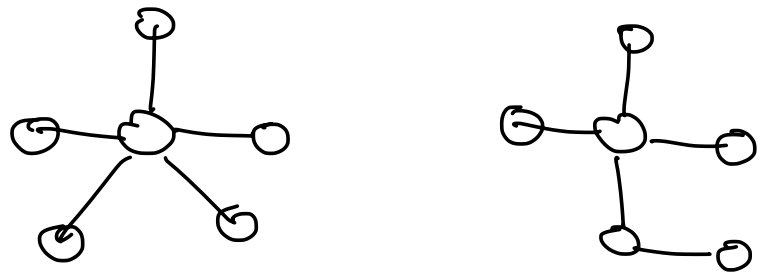
(b) What is the min # of links L of an undirected network of N nodes which is connected?

○ $N+1$ N NONE



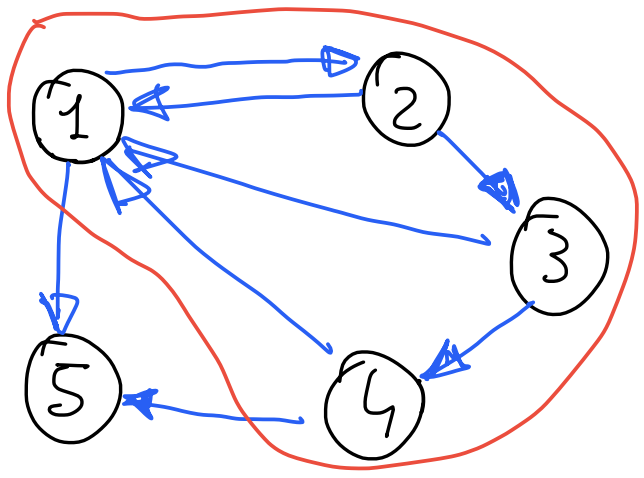
$L = N-1$ links

TREES



Q: CONNECTEDNESS

Consider the network with



$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

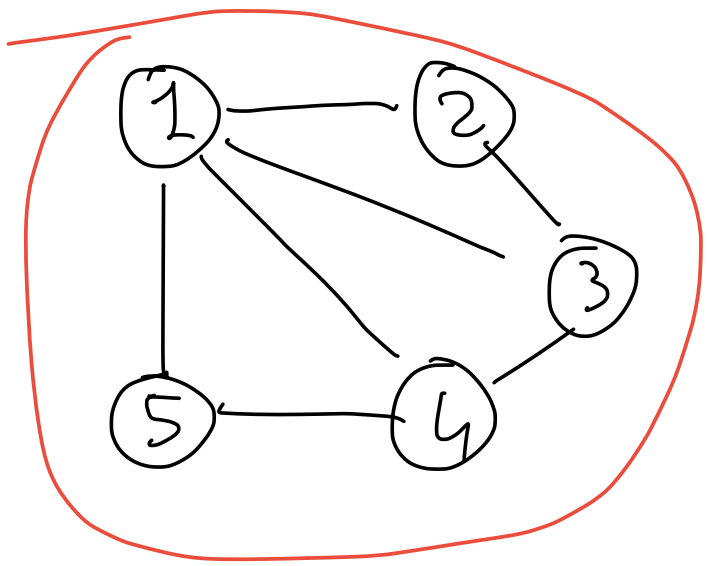
(a) How many SCC ?

0 (1) 2 NONE

$\{1, 2, 3, 4\}$ + 5

(b) How many WCC ?

0 (1) 2 NONE



$\{1, 2, 3, 4, 5\}$